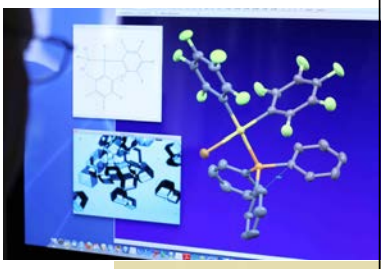
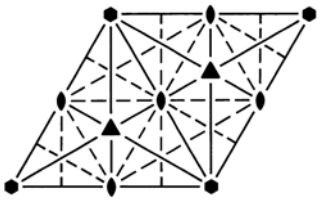
University of Zurich[UZH]

Department of Chemistry

# Fundamentals of Diffraction



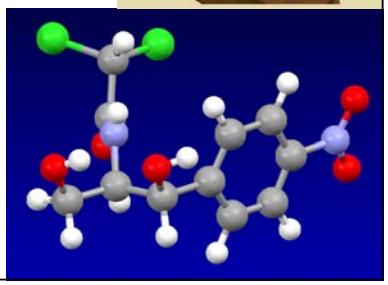


Anthony Linden

7th European Crystallography School

Lisbon

July 10-15, 2022



## Crystallography spans science

- Chemistry, physics, mathematics
- Mineralogy, geology
- Materials science
- Biochemistry
- Biological sciences
- Pharmaceutical sciences
- Medical sciences

# Crystallography

- Chemical crystallography
  (focus on small-molecule single-crystal crystallography)
- Macromolecular crystallography
- Powder diffraction
- Fibre diffraction
- Quasi-crystals
- Aperiodic structures
- Incommensurate structures

# Chemical Crystallography

- Answering chemical questions

We will discuss...

- How to get the most out of a crystal structure
- What we can do
- What we cannot do
- Principles of diffraction – how it works
- Unit cells, crystal systems, lattices
- Symmetry, point groups
- Systematic absences, space groups

# Crystallographic Parameters

| | | | |
|---|---|---|---|
| space group | P-1 | crystal description | orange block |
| $a$, Å | 6.3358(7) | $\theta$ range, deg | 2.94 to 30.45 |
| $b$, Å | 8.6054(9) | index ranges | -8<=h<=9 |
| $c$, Å | 11.6427(13) | | -12<=k<=11 |
| $\alpha$, deg | 91.015(13)° | | -16<=l<=16 |
| $\beta$, deg | 94.877(13)° | total no. of data | 7400 |
| $\gamma$, deg | 102.005(13)° | no. of unique data, $R_{int}$ | 3413,0.0529 |
| $V$, Å$^3$ | 618.25(12) | observed data [a] | 2456 |
| Z | 2 | max. / min. transmission | 0.6116 / 0.4486 |
| $\rho_{calcd}$, g/cm$^3$ | 2.532 | data / restraints / parameters | 3413 / 0 / 155 |
| $\mu$(Mo K$\alpha$), mm$^{-1}$ | 13.059 | goodness of-fit on F$^2$ | 0.927 |
| F(000) | 432 | $R$ [a,b] | 0.0457 |
| crystal size, mm$^3$ | 0.06 x 0.05 x 0.04 | $wR^2$ [a,c] | 0.1090 |
| | | max, min peaks, e/Å$^3$ | 1.756 / -2.130 |

[a] Observation criterion: $I > 2\sigma(I)$. [b] $R = \Sigma||F_o|-|F_c||/\Sigma|F_o|$. [c] $wR^2 = \{\Sigma[w(F_o^2-F_c^2)^2]/\Sigma[w(F_o^2)^2]\}^{1/2}$.

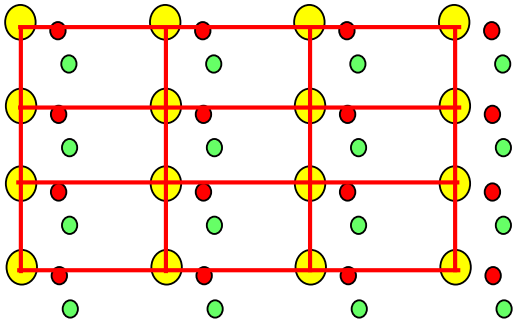
# Why do a crystal structure analysis?

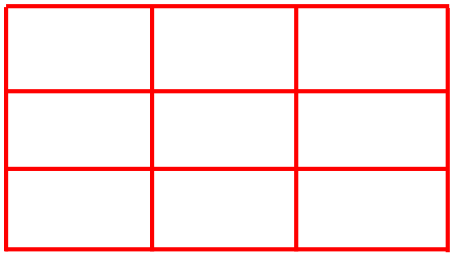
- Understand materials in the solid state
- Chemical and structural identity
- Molecular / structural geometry » electronic structure
- Molecular interactions (H-bonds, C-H···O, etc.)
- Molecular conformation & configuration
- Graphical depiction
- Phase transitions, polymorphism
- Correlation between atomic structure and macroscopic crystal orientation

# Construction of a Crystal

A crystal is a regularly repeating arrangement of atoms, molecules, or some structural motif.

Principal motif





**Crystal structure**

**=**

**Motif**

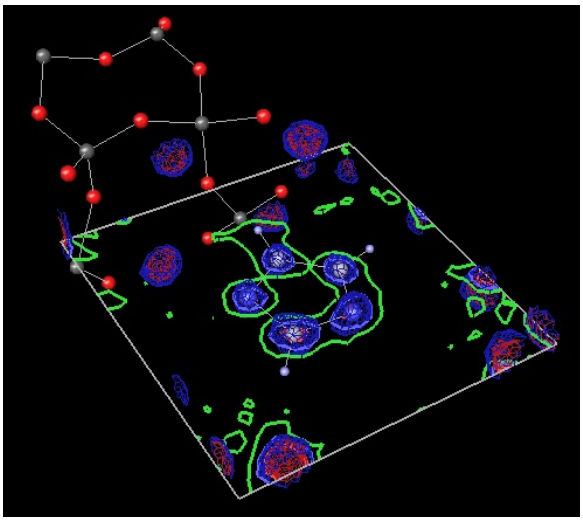
**+**

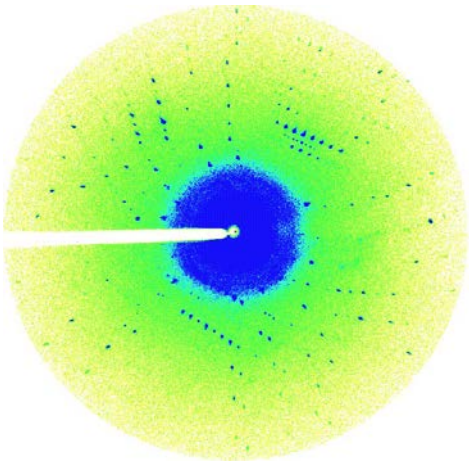
**Lattice**



# Why Crystals?

- The regularly repeating arrangement acts as a tiny diffraction grating.
- The diffraction pattern delivers the most useful information if the crystal is SINGLE and of high quality.



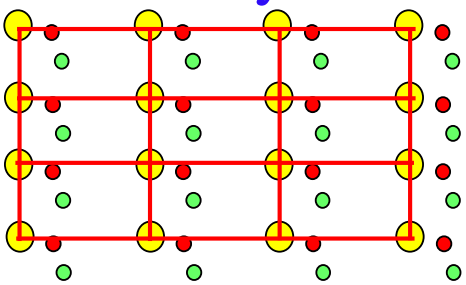


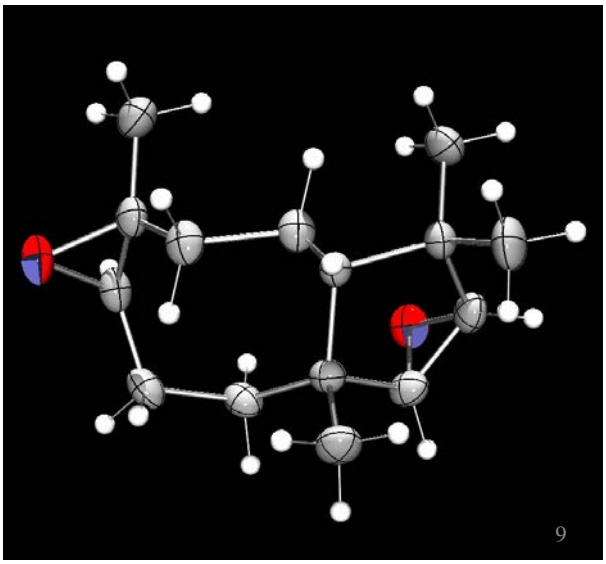
- From the diffraction pattern one can calculate a 3-D map of electron density and so derive the crystal structure (non-trivial).

# X-ray Crystal Structure Analysis

The crystal structure yields…

- 3-D molecular & extended lattice structure.
- The smallest repeating unit is often the chemically interesting moiety - molecule.
- A crystal structure determination is the "definitive" analysis.

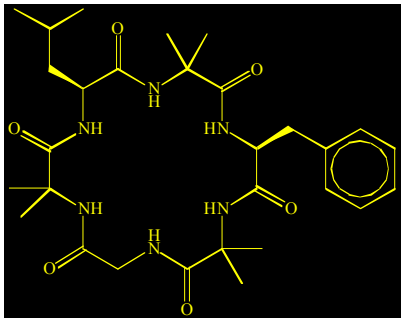





# What am I looking at?

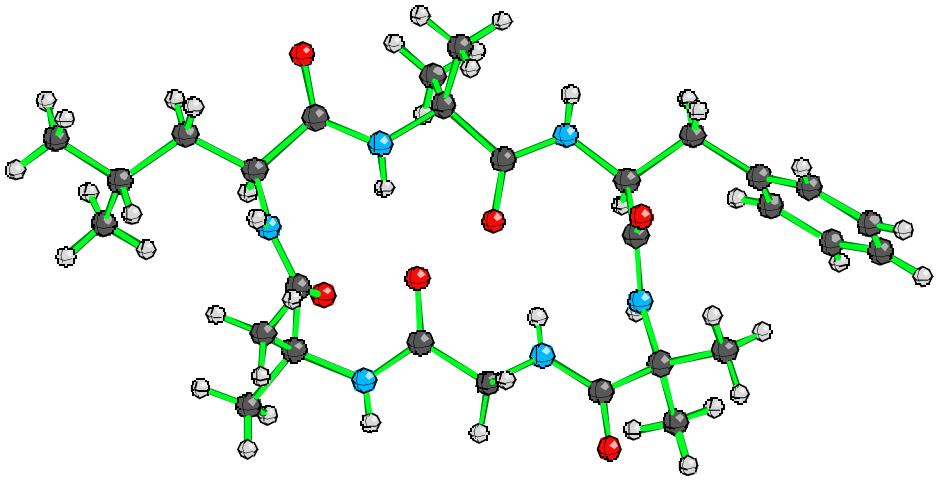
- 3-D electron density map
- You can "see" the molecule
- The "definitive" analysis
- IR, MS, NMR, elemental analyses...
  - give only part of the picture
  - spectra must be interpreted

# Obtain a 3-D image of molecule



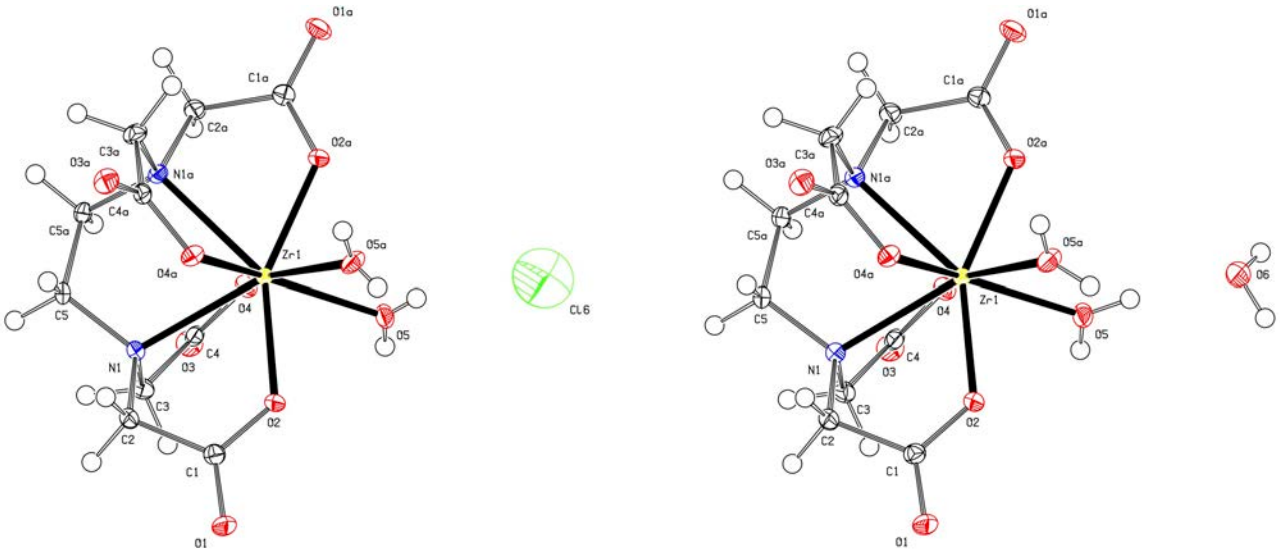

Chemist's 2-D notion...

and reality



# The "definitive" analysis

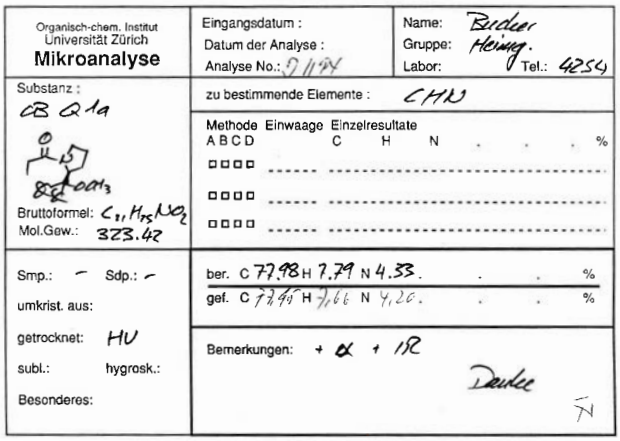
## – when combined with logical chemical reasoning



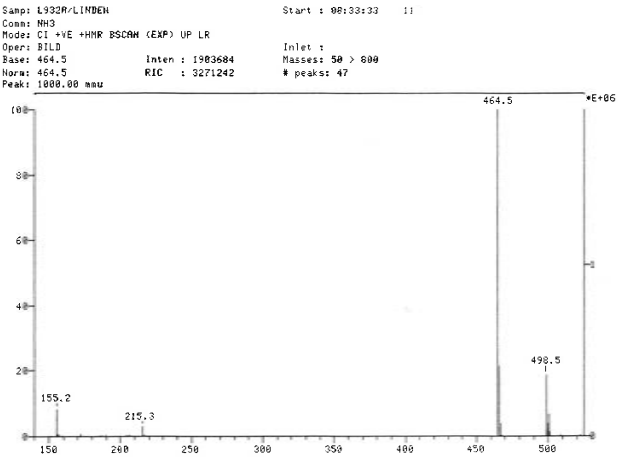

Isolated atom is probably O of $H_2O$ rather than $Cl^-$
(charge balance & displacement ellipsoid size)

# What do we learn from analyses?

- Elemental analysis:
  - ratios of (some) elements
  - might lead to molecular formula
  - not all elements detectable

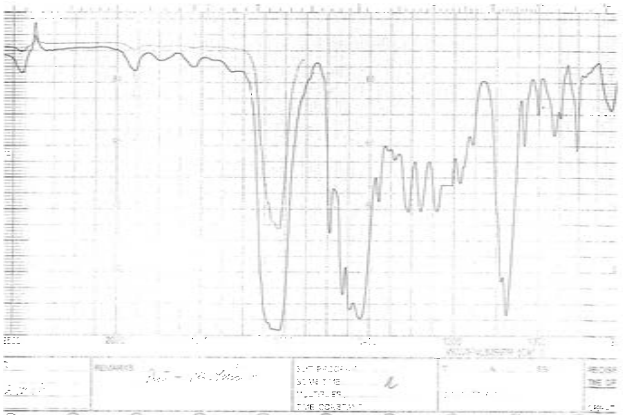



- Mass spectra:
  - molecular formula
  - composition of some fragments
  - not necessarily all connectivity
  - no 3D information

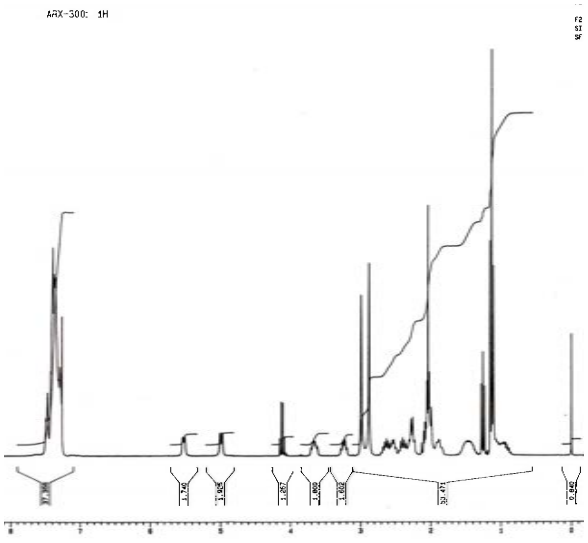



## Spectra interpretation relies on accumulated knowledge

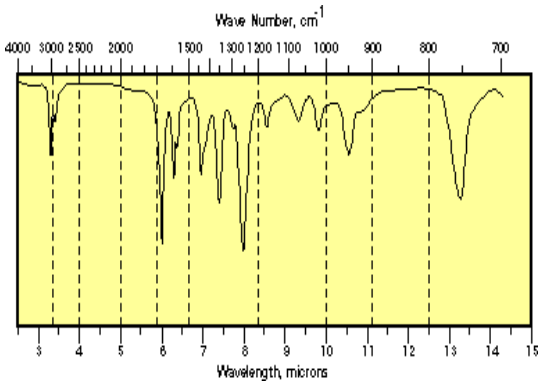
- IR:
  - functional groups
  - deducing connectivity difficult
  - no 3D information
  - not all elements (easily) detectable



- NMR – very powerful:
  - molecular formula
  - full connectivity with care
  - some 3D information

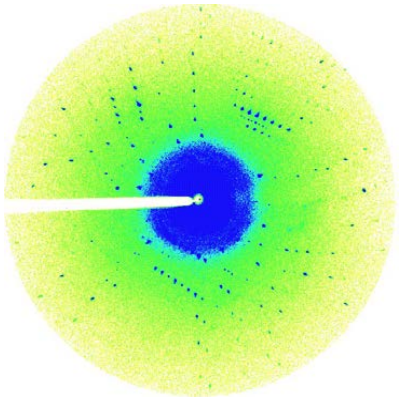


# IR




>3200: OH or NH present
3100: moderate peak suggesting unsaturated CH
2900: weak peak indicating possible saturated CH
2200: no unsymmetrical triple bonds
1690: strong carbonyl absorbance
1610: weak absorbance bands consistent with carbon-carbon double bonds

# X-ray Crystallography

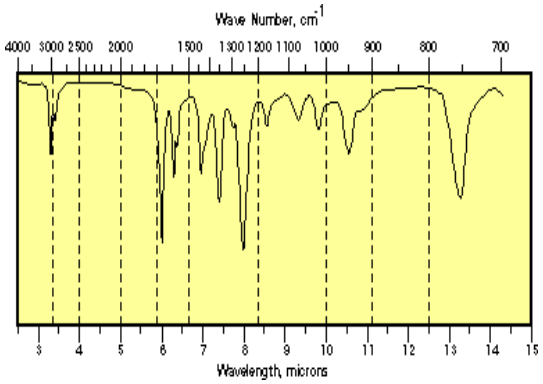


~1000 spotty images

?

Size of diffraction grid
Some symmetry information


# IR

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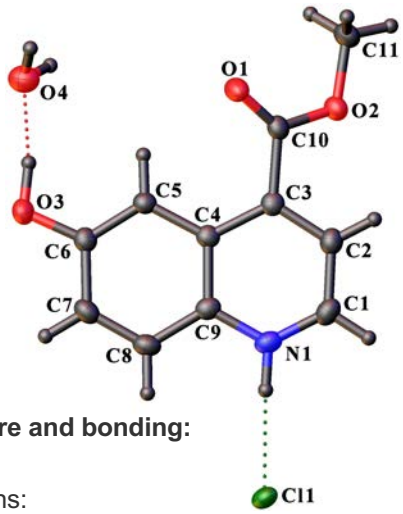



**3D structure and bonding:**

Bond lengths:

| | | | |
|---|---|---|---|
| O1–C10 | 1.226(2) Å | C1–N1 | 1.342(3) Å |
| C3–C10 | 1.503(5) Å | C6–O3 | 1.432(3) Å |

Bond angles:

| | | | |
|---|---|---|---|
| C1–N1–C9 | 121.9 (2)° | C10–O2–C11 | 122.4(2)° |

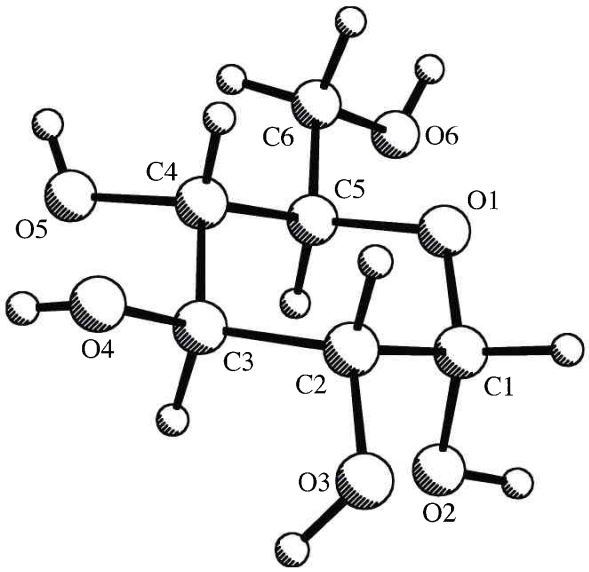
Absolute configuration  Intermolecular interactions

# Crystal structure determination

- When done carefully under the right conditions with a suitable quality crystal, it usually unambiguously gives:

  - full elemental composition
  - full connectivity
  - full 3D geometry and conformation
  - absolute configuration for enantiomerically pure chiral molecules
  - intermolecular interactions and packing of molecules in the solid state

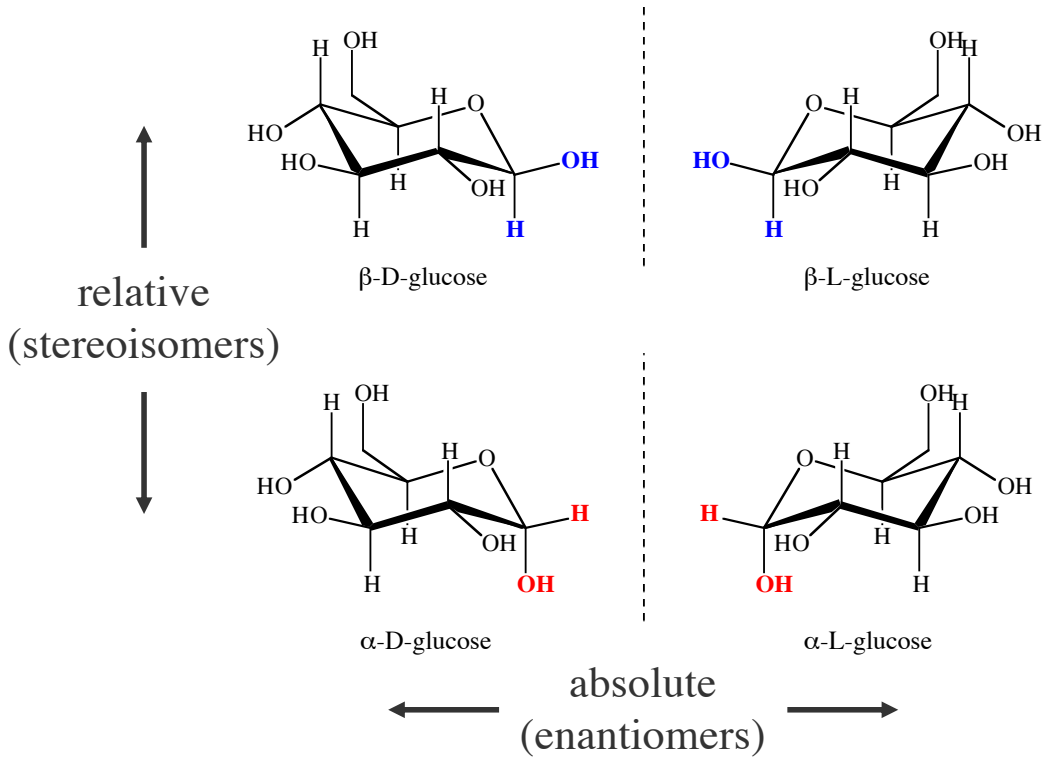
  Isolated molecules or those in solution might have a different conformation




# Advantages

- Complete 3-D structure
- Relative configuration
- Absolute configuration (sometimes)
- Intermolecular interactions
  - hydrogen bonding

# Relative and absolute configuration



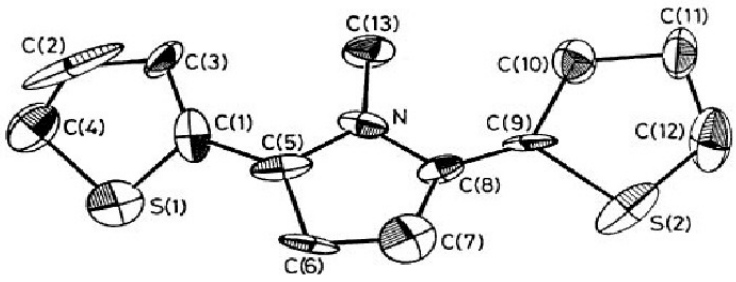

# Disadvantages

- Structure represents solid-state conformation
  - solution structure might be different
- Crystal growth
- Crystal not always representative of the bulk batch
- Purity is important, otherwise ambiguity remains
- If solution contains a mixture, what crystallises first (as a nice crystal), a crystal containing one component or the mixture (what ratio of components)? Solid solutions rare.
- A high *ee* mixture might crystallise as a racemic crystal.

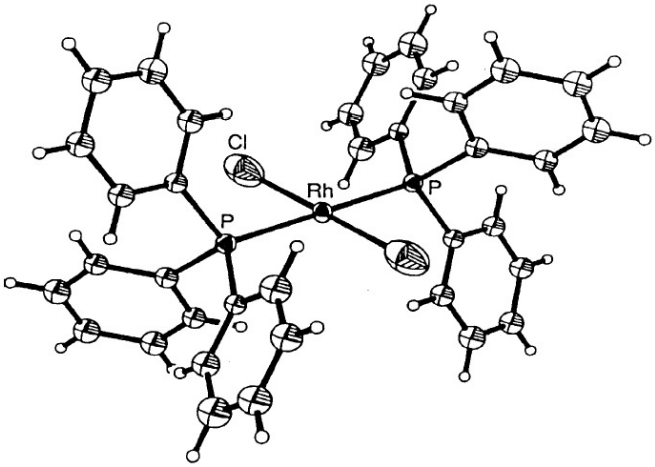
# Philosophy

- A job worth doing is worth doing as well as possible
- Get the best data the first time
- **Garbage In ⇒ Garbage Out**
- Poor results ⇒ ambiguity remains or unpublishable
- Might be hard to repeat work months/years later
- A little extra effort early on for crystal growth can be rewarding

# Examples of Bad Structures




*Chem. Comm.* **1989**, 1318.

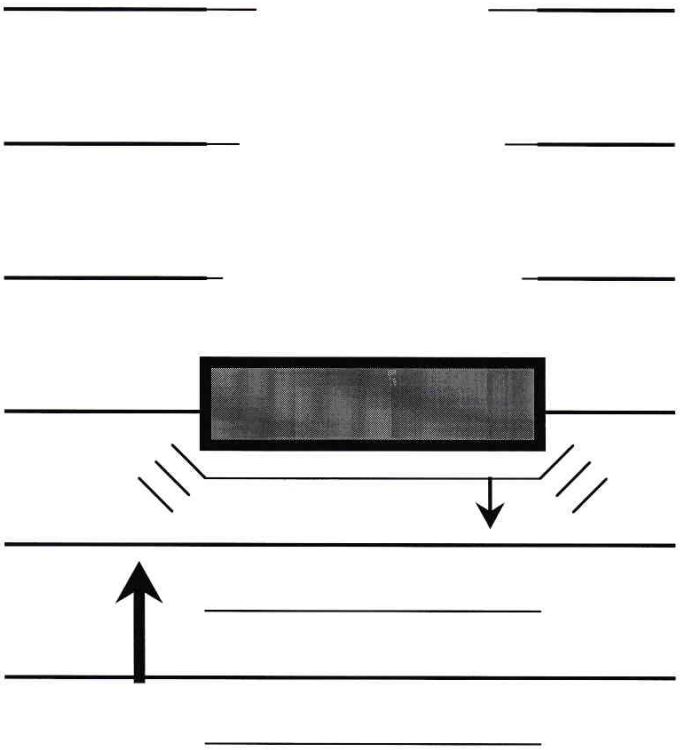



*Chem. Comm.* **1990**, 1733.

Examples taken from:

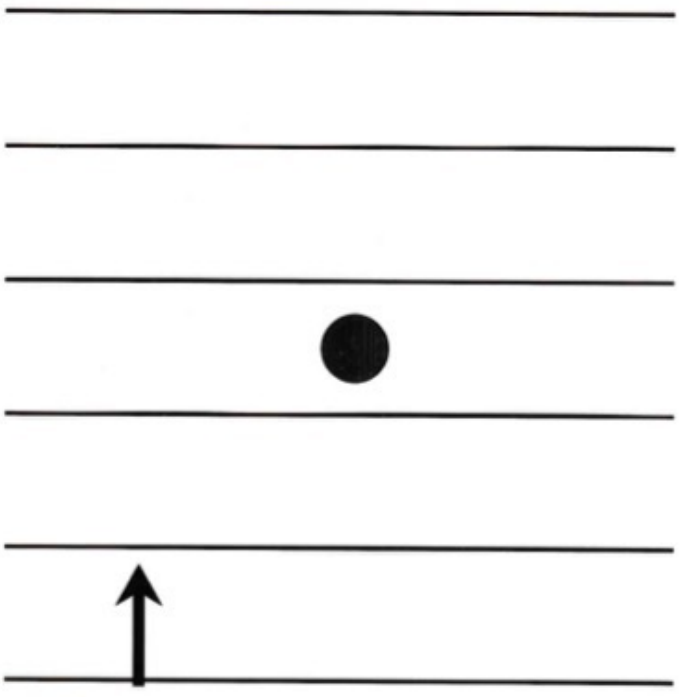
Troublesome Crystal Structures
*J. Res. Nat. Inst. Stan. Technol.* **1996**, 327.

# How do we see things?



- We only detect light (photons) scattered from objects
- We only sense the information in the scattered light (waves)
- Does all scattered light contain information about everything it passes?
- In this case with a large object, yes

# What about small objects?



- The waves contain very little information about objects they pass when the wavelength is larger than the object

# Why use X-rays?

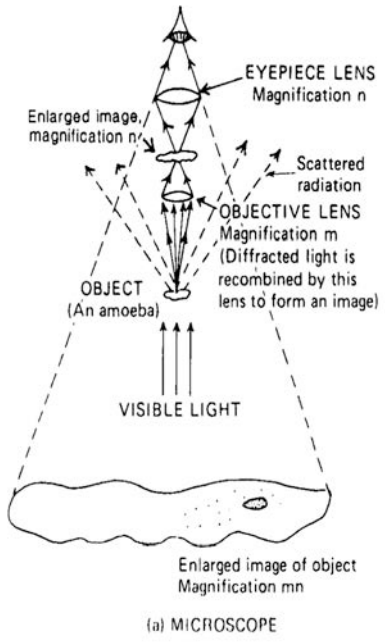
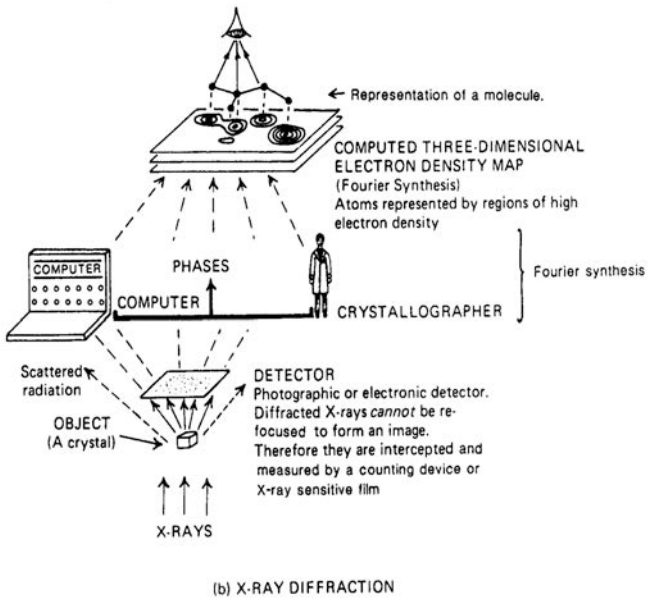



**FIGURE 1.1 Analogies Between Light Microscopy and X-ray Diffraction.**

- Atoms in molecules are separated by 0.1-0.3 nm
- To view very small objects (< $10^{-7}$ m) need shorter wavelength than visible light
- X-rays have a wavelength ~0.1 nm.
- Why not use an X-ray microscope?
- Cannot refract X-rays easily

Glusker & Trueblood: Crystal Structure Analysis – a Primer

# To "view" atoms



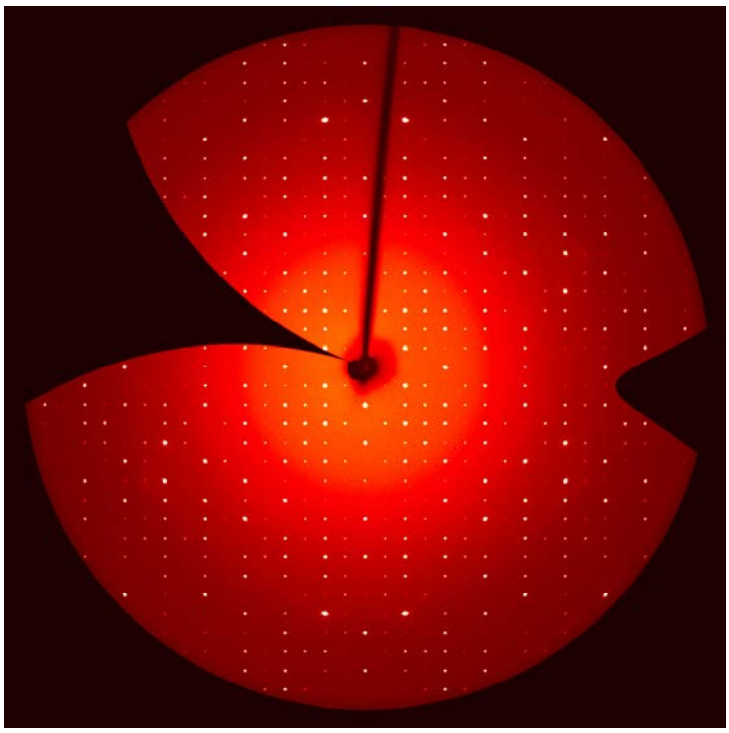

- Record scattered, in particular diffracted, X-rays
- Use a computer as the "lens"

Glusker & Trueblood: Crystal Structure Analysis – a Primer

How can we use the diffraction pattern to deduce the form and orientation of the "objects" in the crystal, i.e. solve the crystal structure?

Reconstructing (unwarping) layers through reciprocal space may be enlightening, *e.g. h*0*l*, *hk*0, 0*kl, h*1*l, etc*.

Highly recommended after ALL data collections



# Key information for success

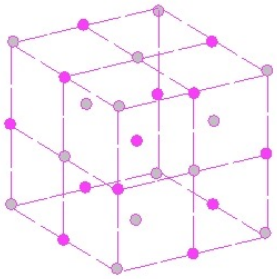
- Size of repeating unit
  - Related to the spacing in the diffraction pattern
- Arrangement of contents of the unit
  - Related to intensities of diffracted beams

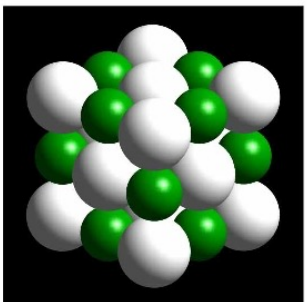
# The Phase Problem

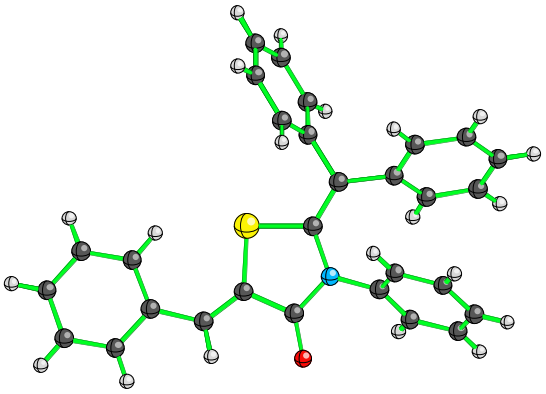
Electron density: $\rho(xyz) = \left(\frac{1}{V}\right)\sum_h\sum_k\sum_l \{|F(hkl)|\cos[2\pi(hx + ky + lz) - \phi(hkl)]\}$

Intensities, ~ $F(hkl)^2$, can be measured

Phases, $\phi(hkl)$, cannot be measured









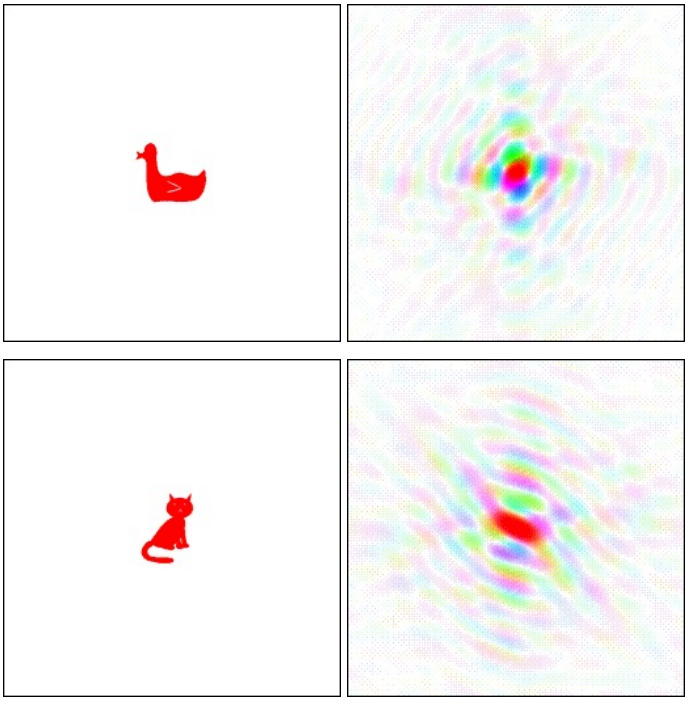
# Intensities and Phases

A duck and a cat and their *Fourier* transforms (diffraction pattern)

Brightness = intensity

Colour = phase

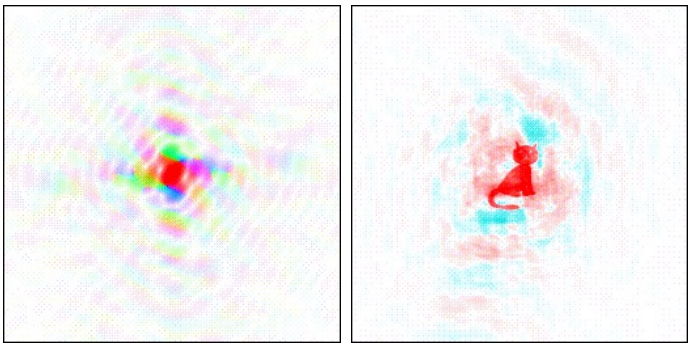
If we know intensity & phase, we can do back transformation



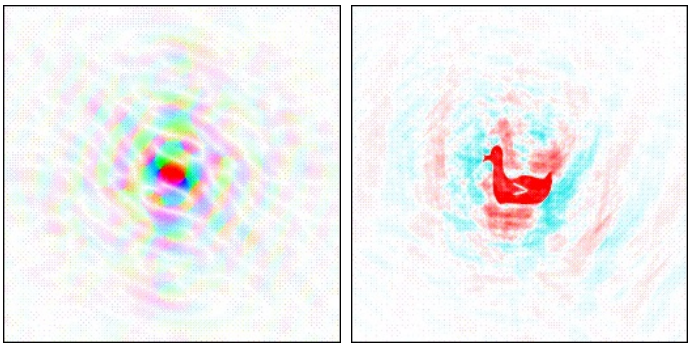
Kevin Cowtan: http://www.ysbl.york.ac.uk/~cowtan/fourier/fourier.html

# The Importance of Phases

Phases from cat, intensities from duck:



Phases from duck, intensities from cat:



The phases have much more information than the intensities.

Kevin Cowtan: http://www.ysbl.york.ac.uk/~cowtan/fourier/fourier.html

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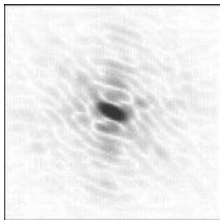
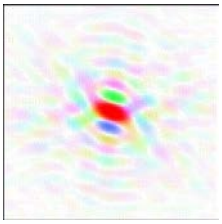
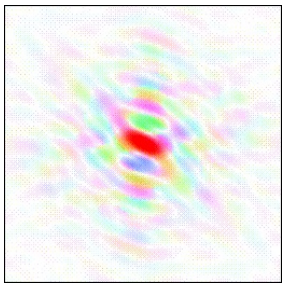
Intensities only; cannot generate an unknown object:

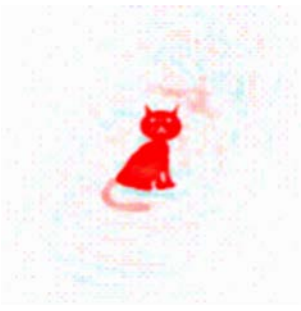



Take a similar known object (Manx cat) from which phases can be calculated:



Manx phases + real intensities. When phases nearly correct, the missing (wrong) parts of the model appear:





How do we arrive at an approximately correct starting model?

Kevin Cowtan: http://www.ysbl.york.ac.uk/~cowtan/fourier/fourier.html

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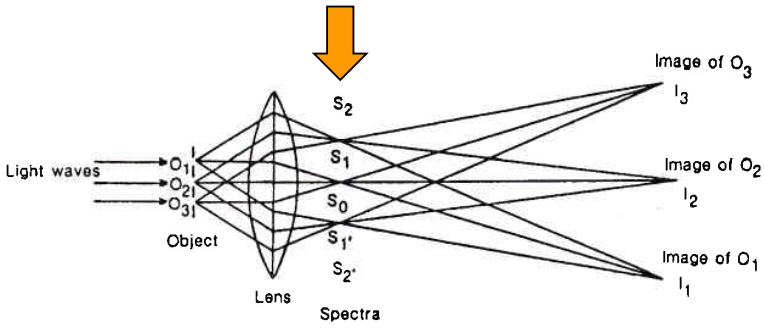
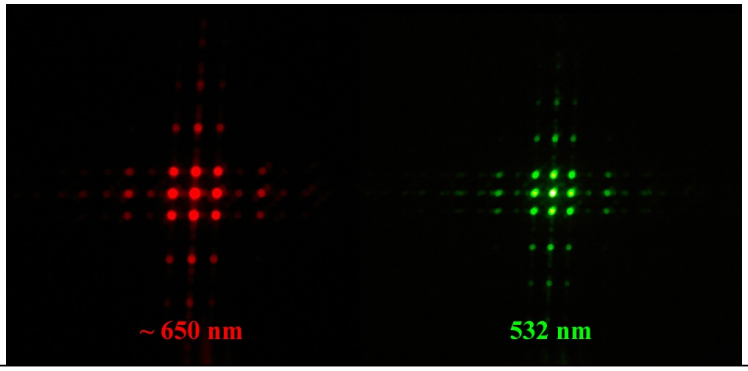
The periodic nature of the structure in the crystal causes diffraction; acts as the “lens”.

Each diffracted beam contains information about every point in the crystal.

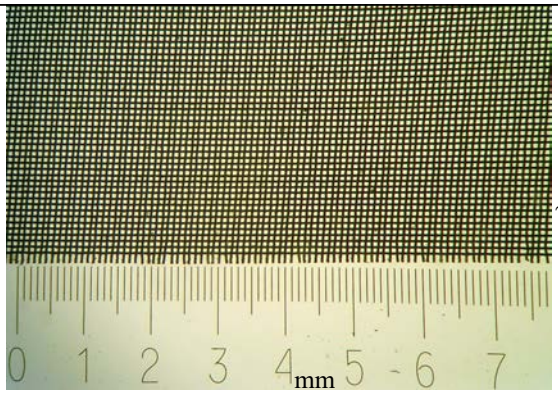
Each point in the crystal contributes to every diffracted intensity in the diffraction pattern.

One is the Fourier transform of the other.

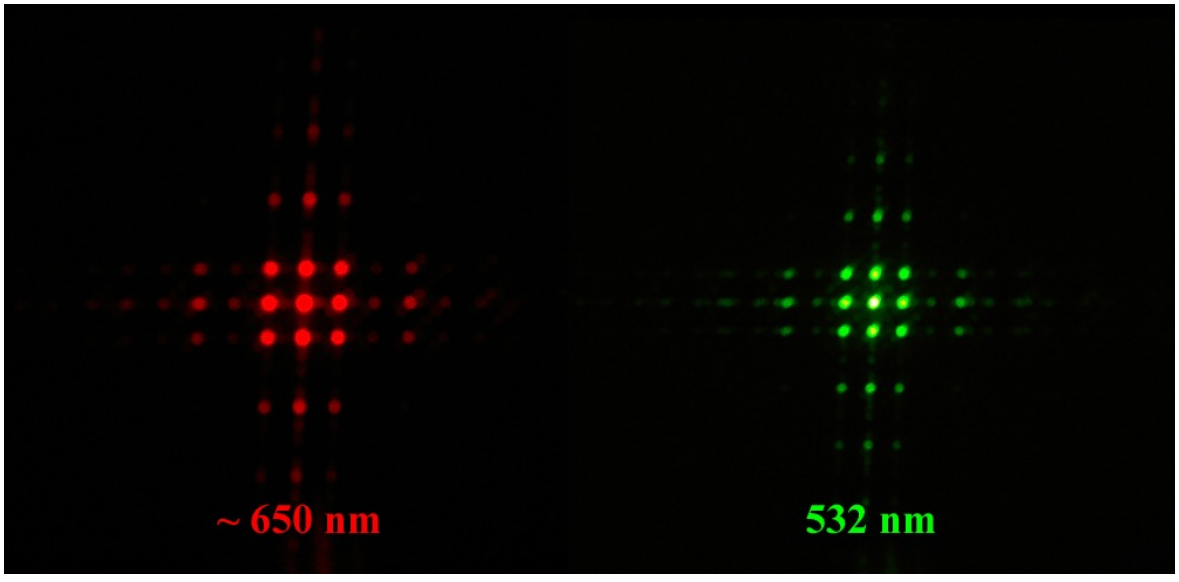


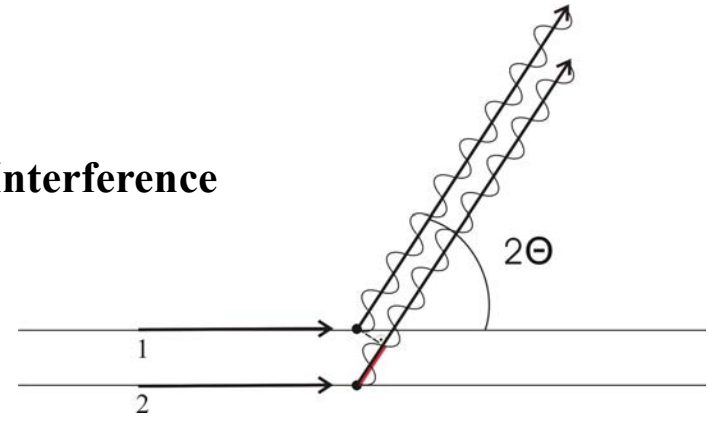

# Diffraction Patterns



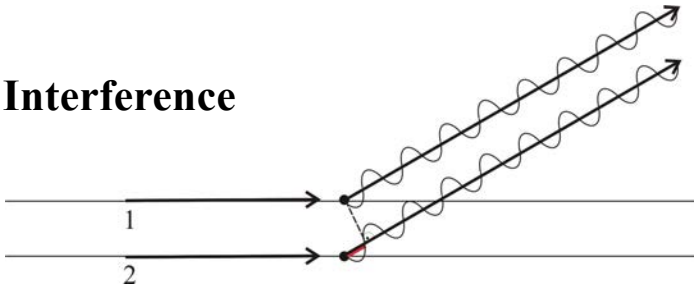

~ 0.1 mm mesh



**Constructive Interference**




**Destructive Interference**




# Diffraction by a single slit

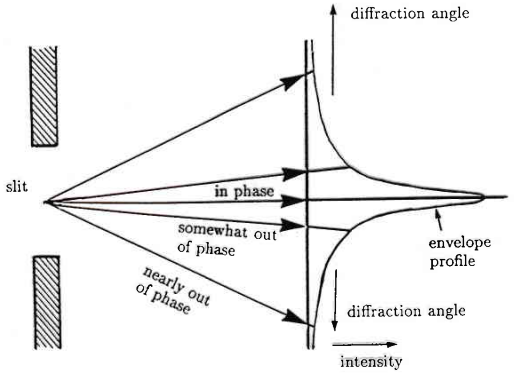



FIGURE 3.5. An illustration of the reason that the diffraction pattern of a single slit has width to its profile (the envelope profile, compare with Figure 3.4). This is because of varying degrees of interference between waves traveling in the directions indicated in this figure. Waves traveling in the direction of the direct beam (if the incident light is perpendicular to the slit) are in phase and give a maximum intensity. At angles other than that of the direct incident beam, the scattered waves are more out of phase. Therefore their intensities decrease because of increased interference. The result is a diffraction pattern with the envelope profile shown here.

In a crystal, this is equivalent to diffraction by the single unit (one or more atoms or molecules) that forms the smallest translationally periodic repeat unit of the crystal

## Diffraction by a single slit

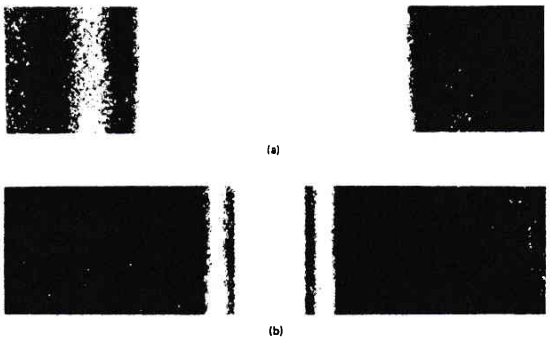



FIGURE 3.1 **Diffraction Patterns of Single Narrow Slits.**

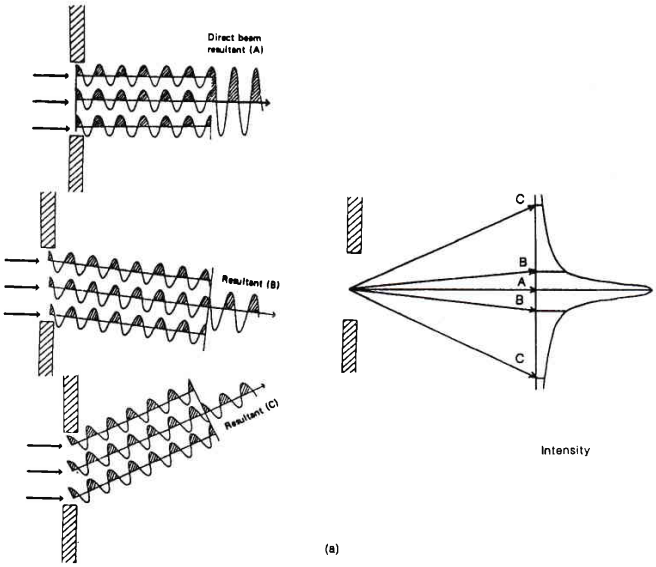
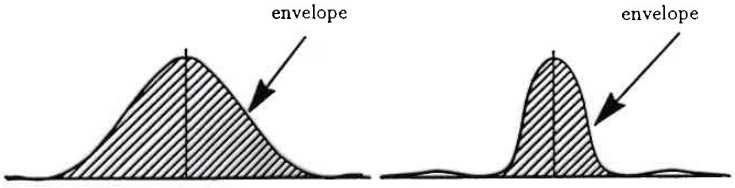


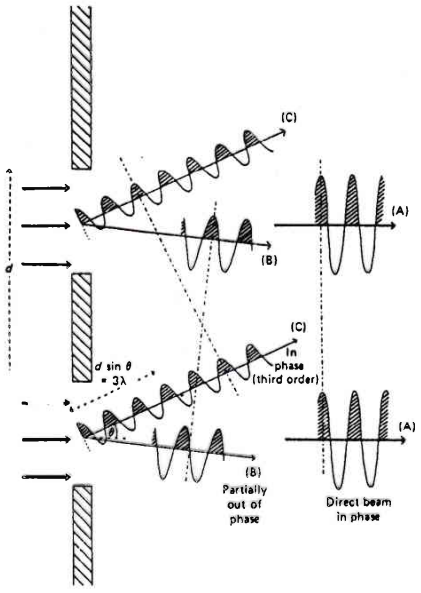

FIGURE 3.3 **Diffraction as the Interference of Waves.**

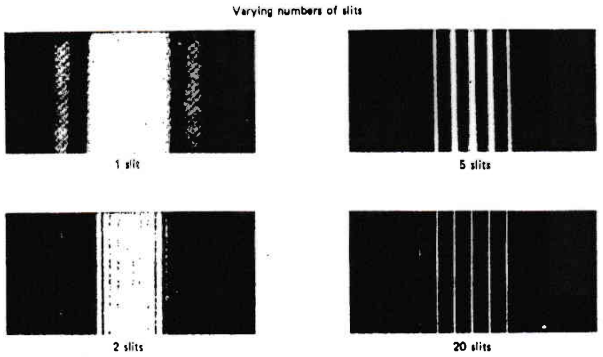



(a) narrow slit (b) wide slit

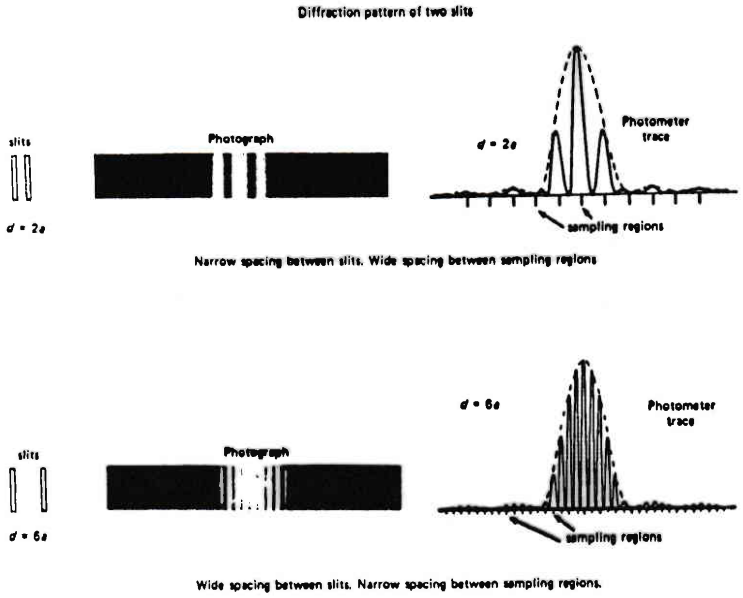
FIGURE 3.4. Scattering by a single slit. (a) Diffraction by a narrow slit and (b) the diffraction pattern of a slit that is wider than that in (a). In both cases the intensity variation shown is referred to as the "envelope." The zero point of the horizontal axis represents the direction of the direct beam (cf. Figure 3.5).

## Diffraction by two slits



# Wide slit spacing, narrow diffraction pattern




- The size and shape of the "envelope" are determined by the diffraction pattern of a single slit.
- The positions and frequency of the regions in which the envelope is sampled are determined by the spacings between the slits.
- The intensity in each sampled region is influenced by the shape of the envelope.

# Wide slit spacing, narrow diffraction pattern

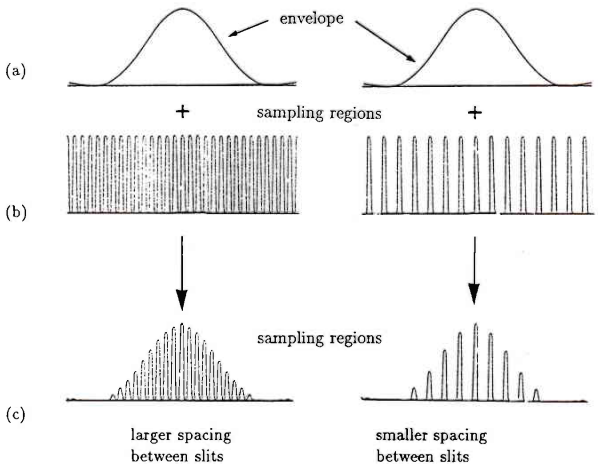
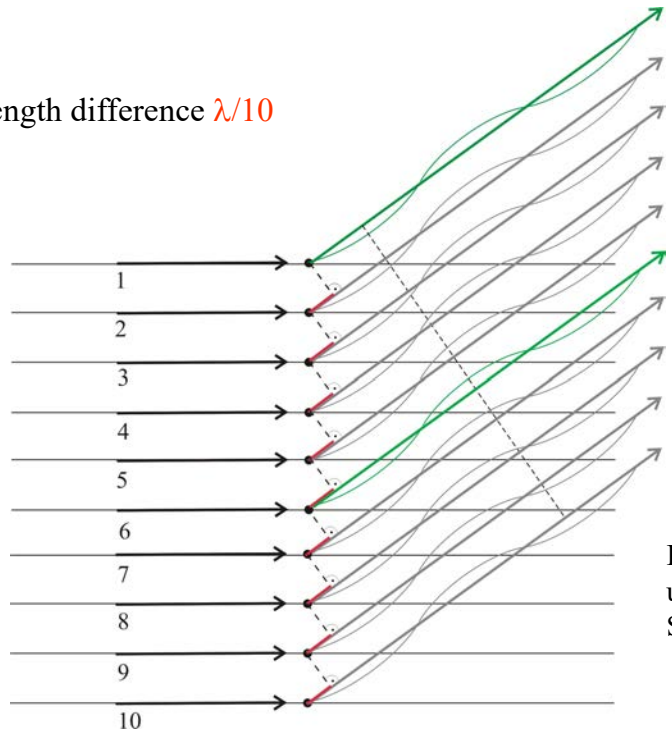



FIGURE 3.6. The diffraction patterns of series of slits. Shown in (a) is the diffraction pattern of a single slit. The sampling regions (from the grating periodicity) are shown in (b). The diffraction pattern is the combination of the envelope profile and the sampling regions, that is, the sampling of the envelope profile at sampling regions only. The results for a series of parallel, regularly spaced slits are shown in (c). The spacing between the slits is greater in the diagrams on the left than in those on the right.

- The diffraction pattern is increasingly sharp, the greater the number of slits.
- Inverse effect.
  Caused by size of one slit
  (= the contents of one repeating unit (envelope))
  and the spacing between the slits
  (= the size of the repeating unit)
- Real vs. reciprocal space
  (slits or real structure vs. diffraction pattern)

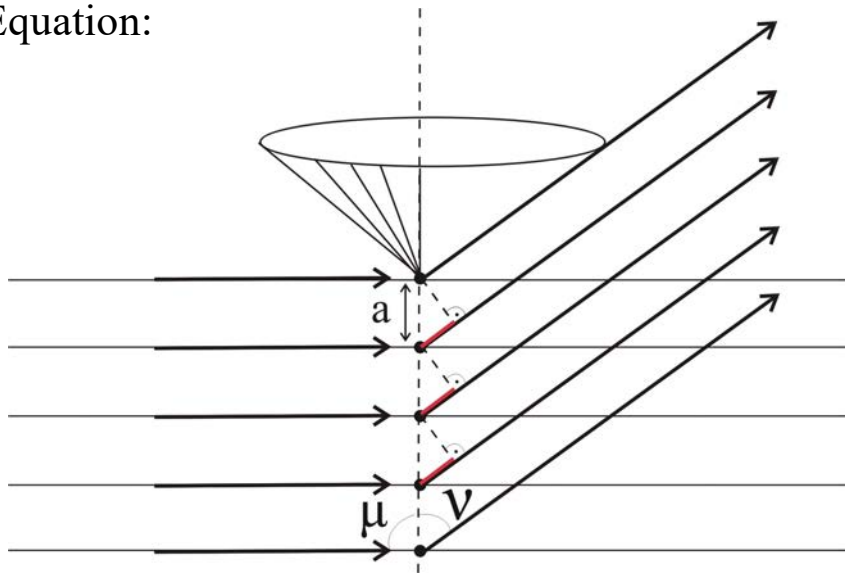
What happens when we are somewhere between complete constructive and destructive interference? i.e. a path length difference between nλ und nλ/2?

e.g. path length difference λ/10




In a crystal there are ca. $10^5$ unit cells along a side.
Sharp reflections result!

Laue-Equation:




**Path length difference: $a \cos\mu_a + a \cos\nu_a = n_1\lambda$**

**For a given incident angle, μ, there is an exactly defined diffraction angle, ν, at which the diffracted beam can be observed for every order of diffraction n = 1,2,3,...**

**Analogous equations for all three directions in space:**

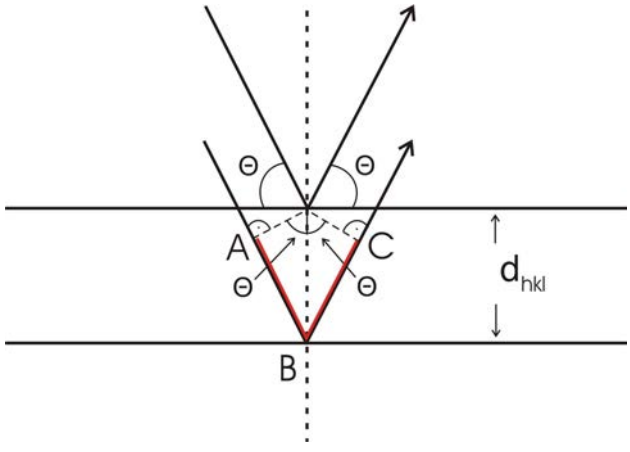
$$a\cos\mu_a + a\cos\nu_a = n_1\lambda$$

$$b\cos\mu_b + b\cos\nu_b = n_2\lambda$$

$$c\cos\mu_c + c\cos\nu_c = n_3\lambda$$

**Diffraction only occurs when all three equations are fulfilled simultaneously, i.e. when the three diffraction cones have a common line of intersection!**

# Bragg Equation (1912):



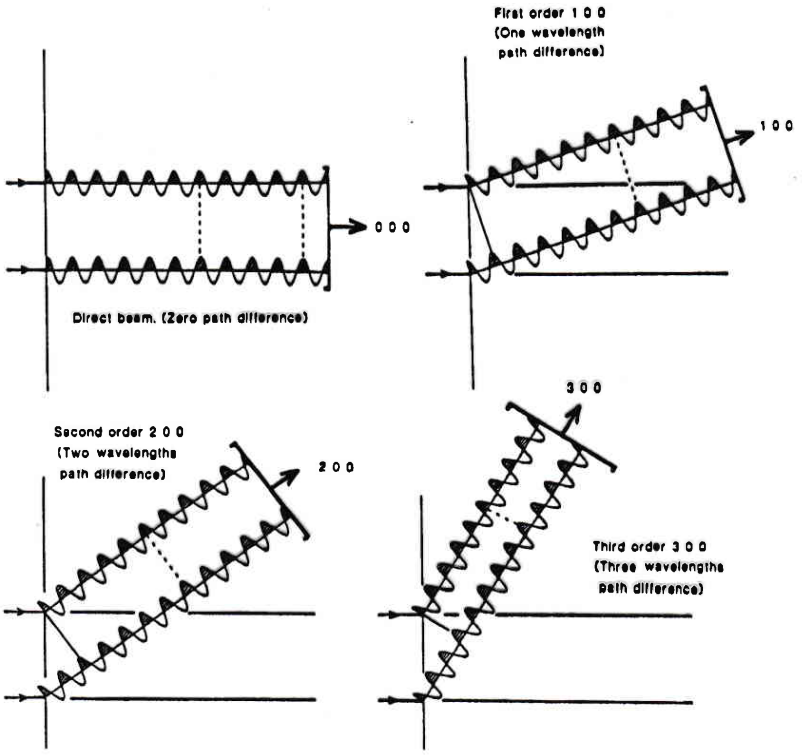

Constructive interference occurs when:

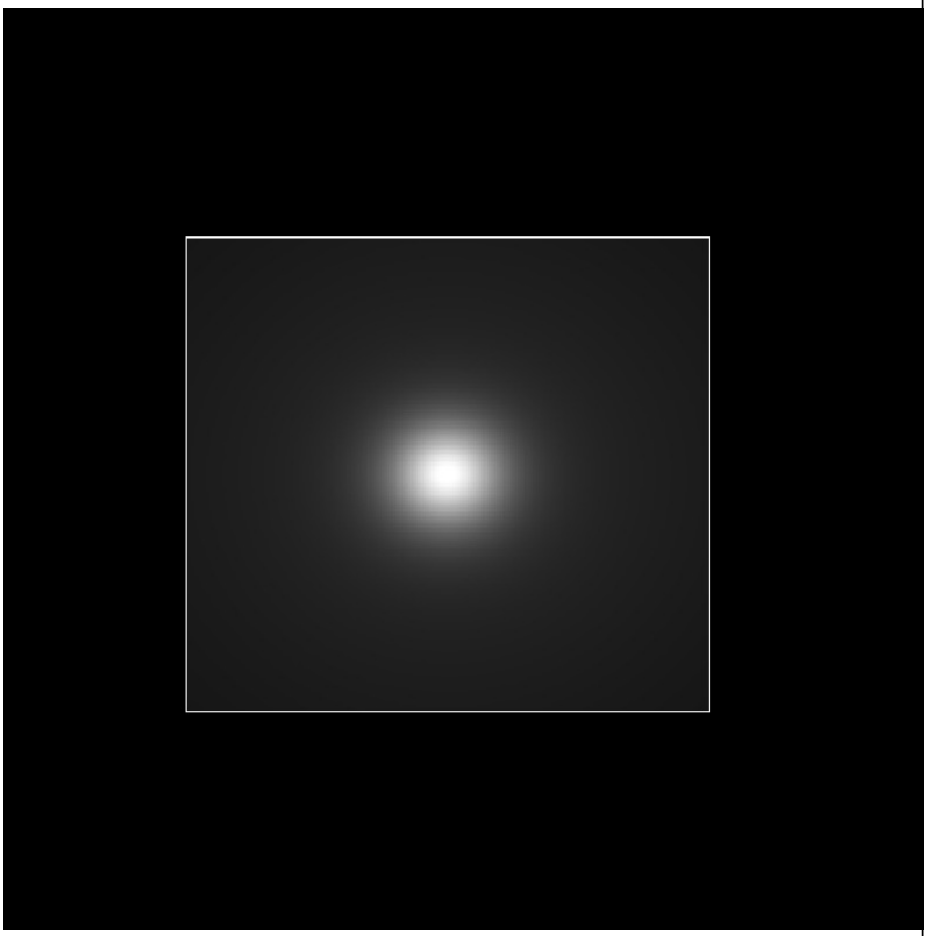
$\Delta = n\cdot\lambda$ ($\Delta$= AB+BC)

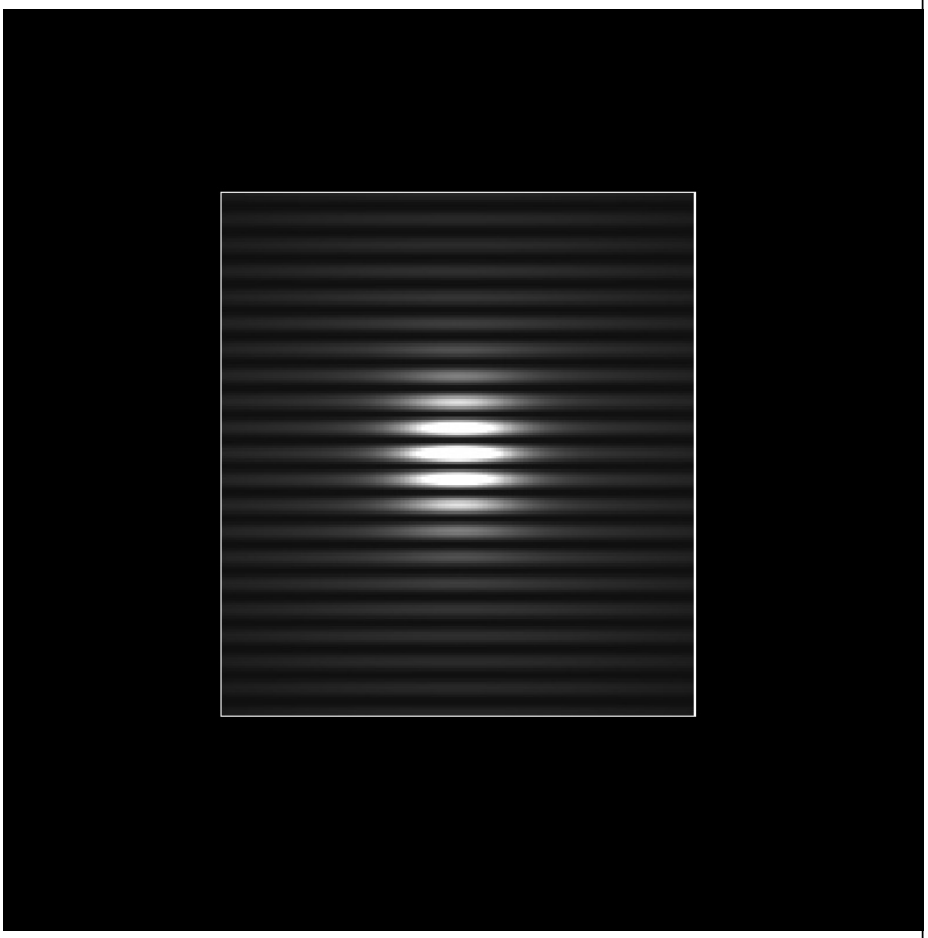
Using:

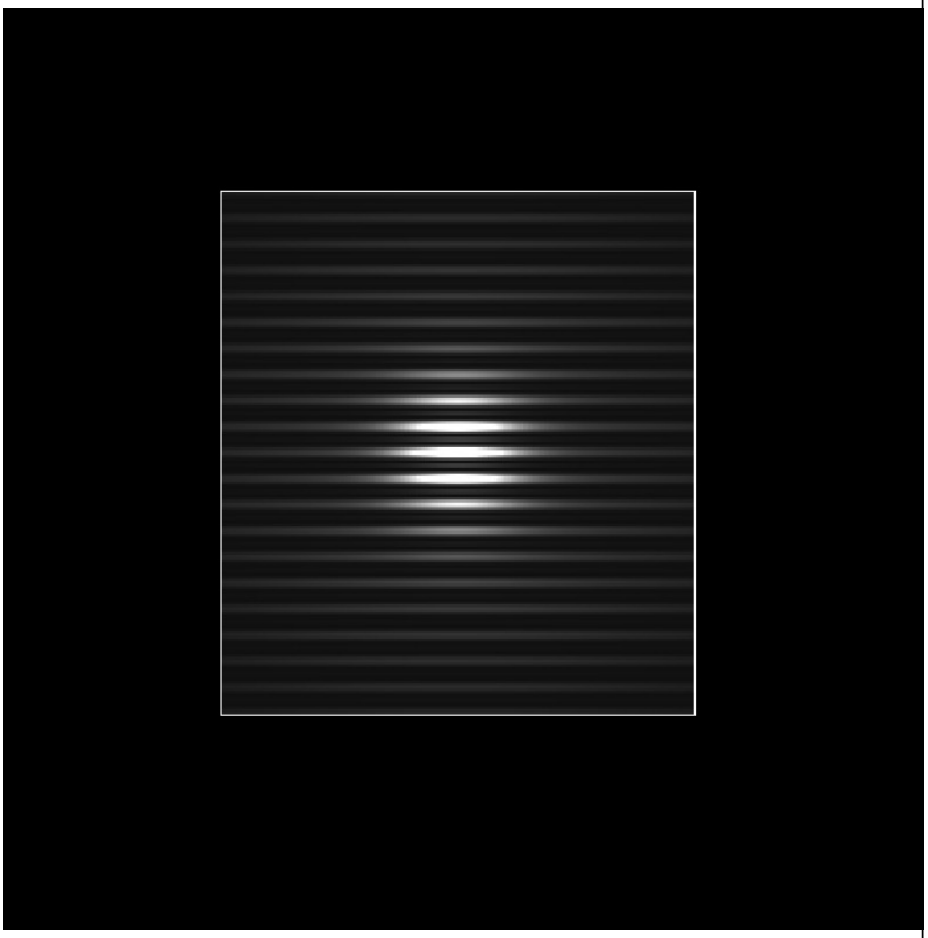
$\sin\theta = (\Delta/2)/d$ results in $n\cdot\lambda = 2d\cdot\sin\theta$

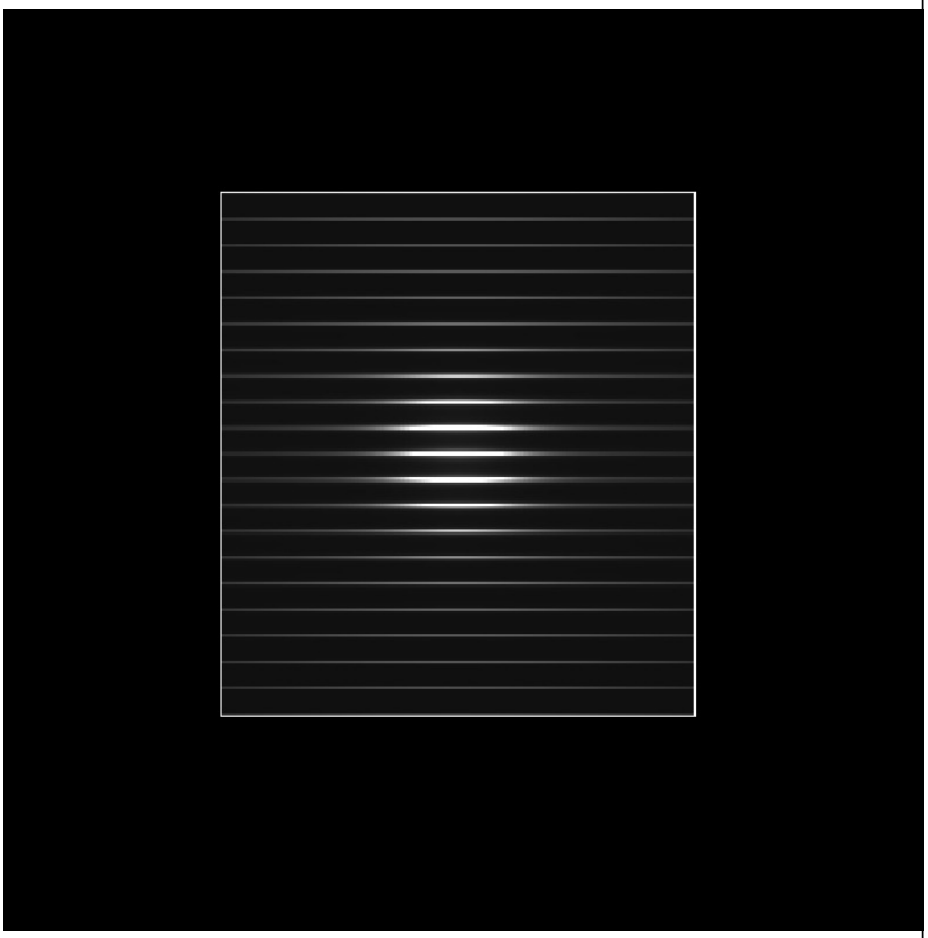
# Orders of diffraction

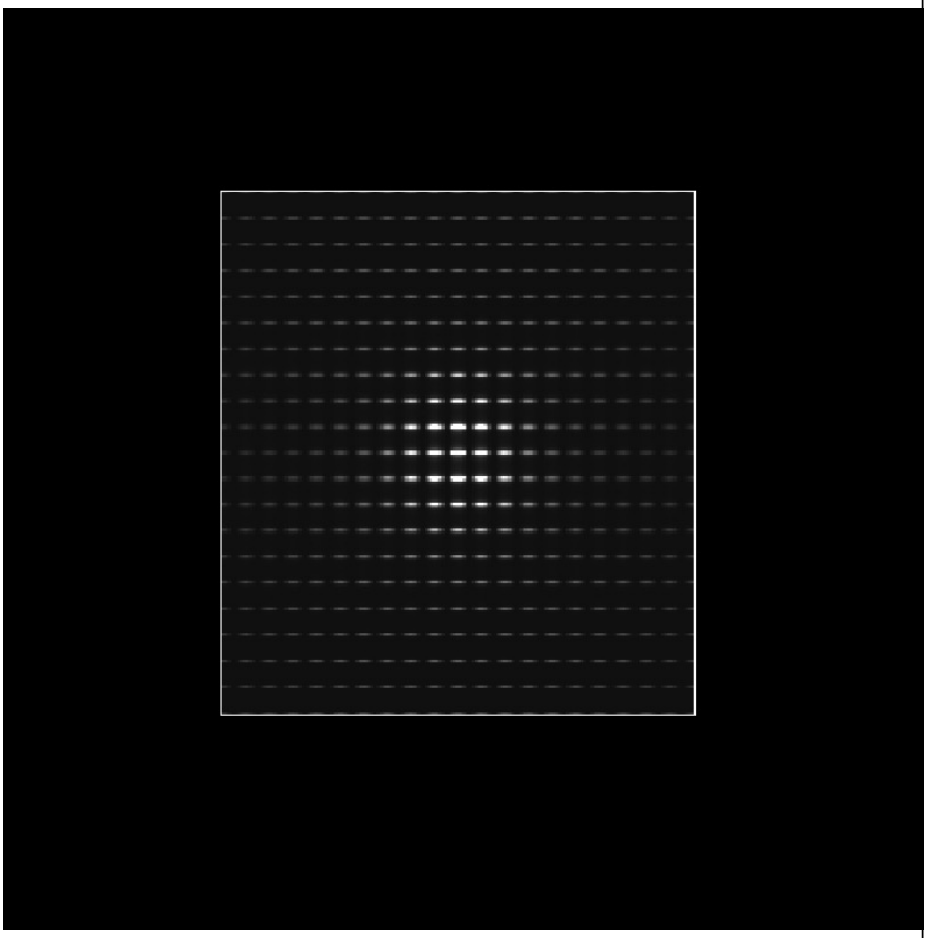


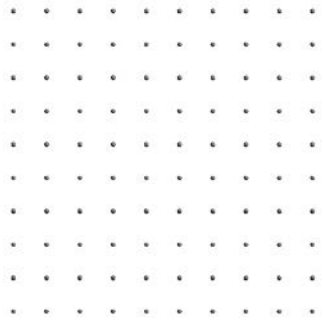



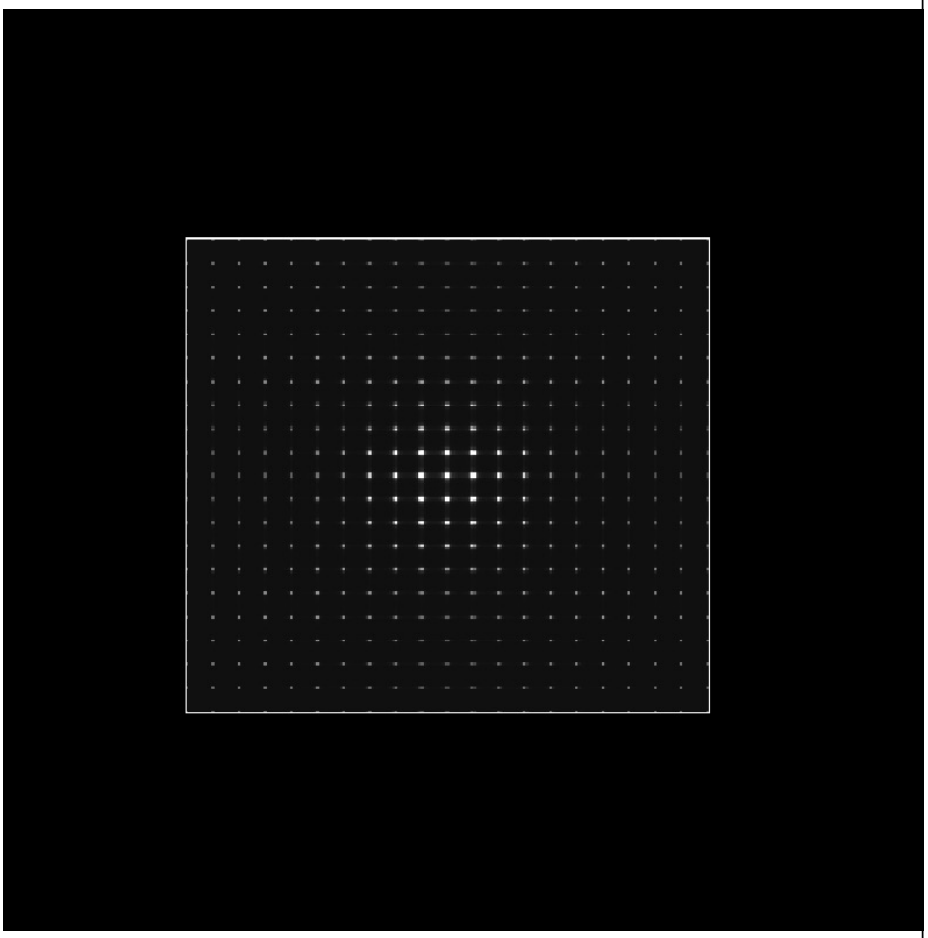


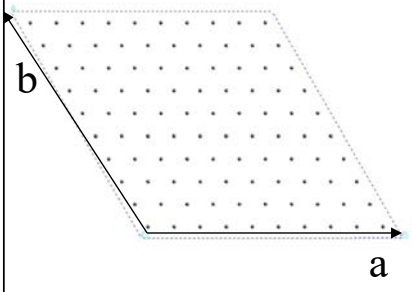


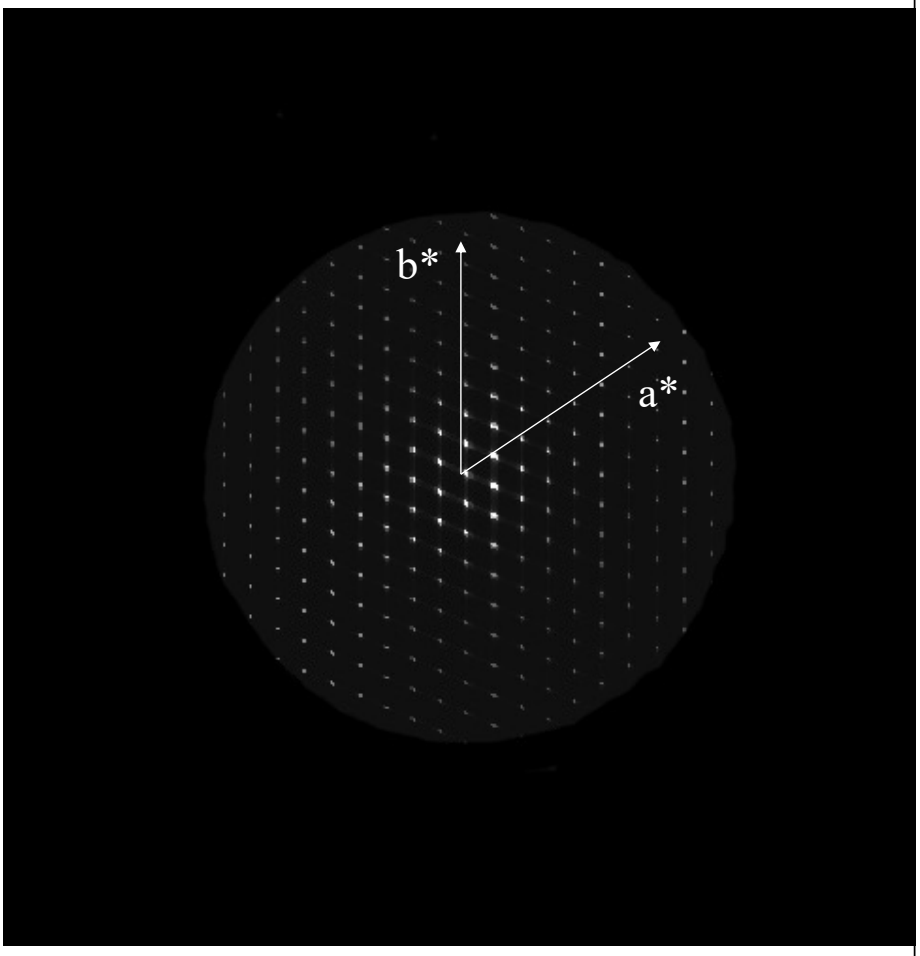






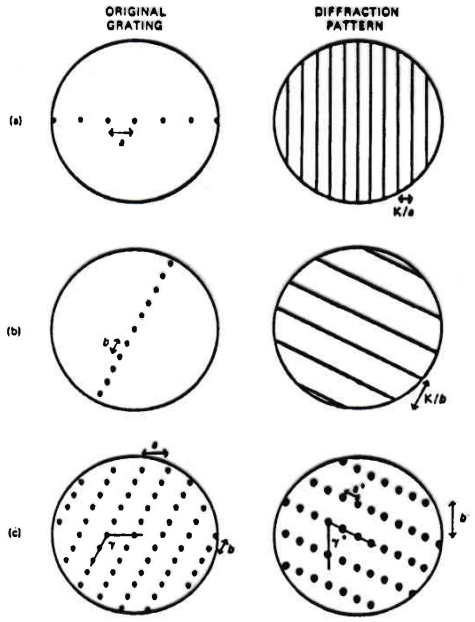


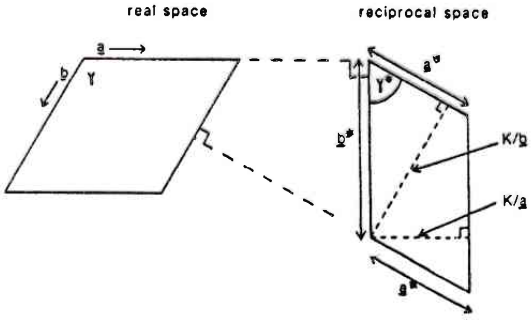

b
a


b*
a*

## 2-Dimensional Diffraction Gratings

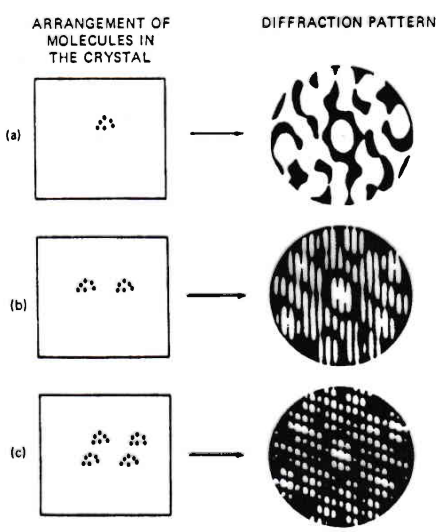
**Original grating** **Diffraction pattern**

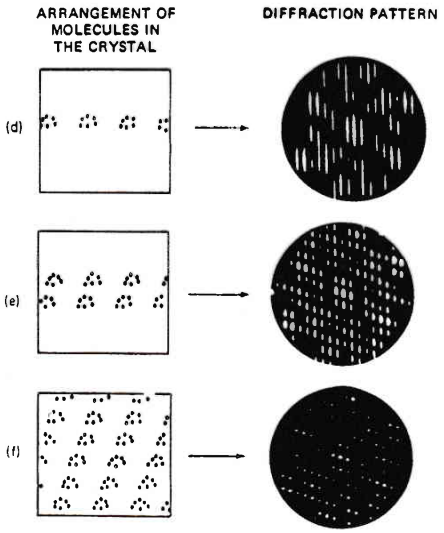


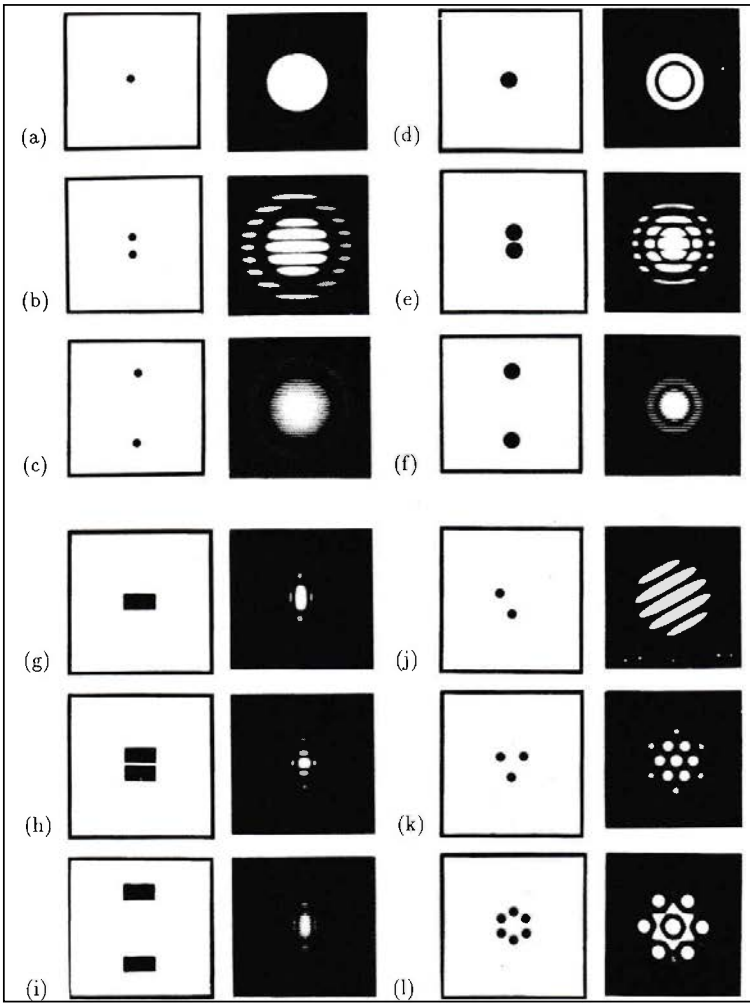




## Diffraction by Molecules

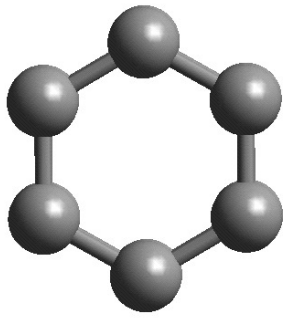
Size of repeating unit affects spacing of pattern.
Contents of unit affects intensities within the pattern.

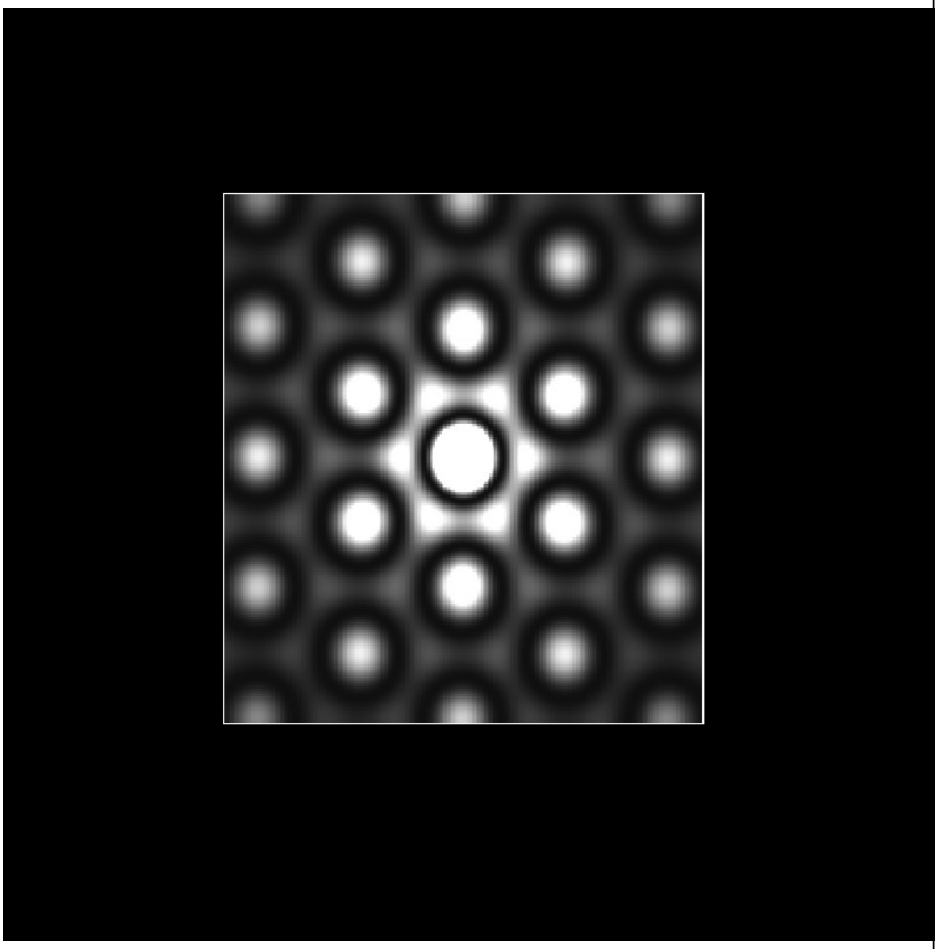


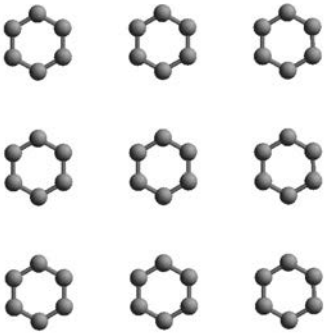



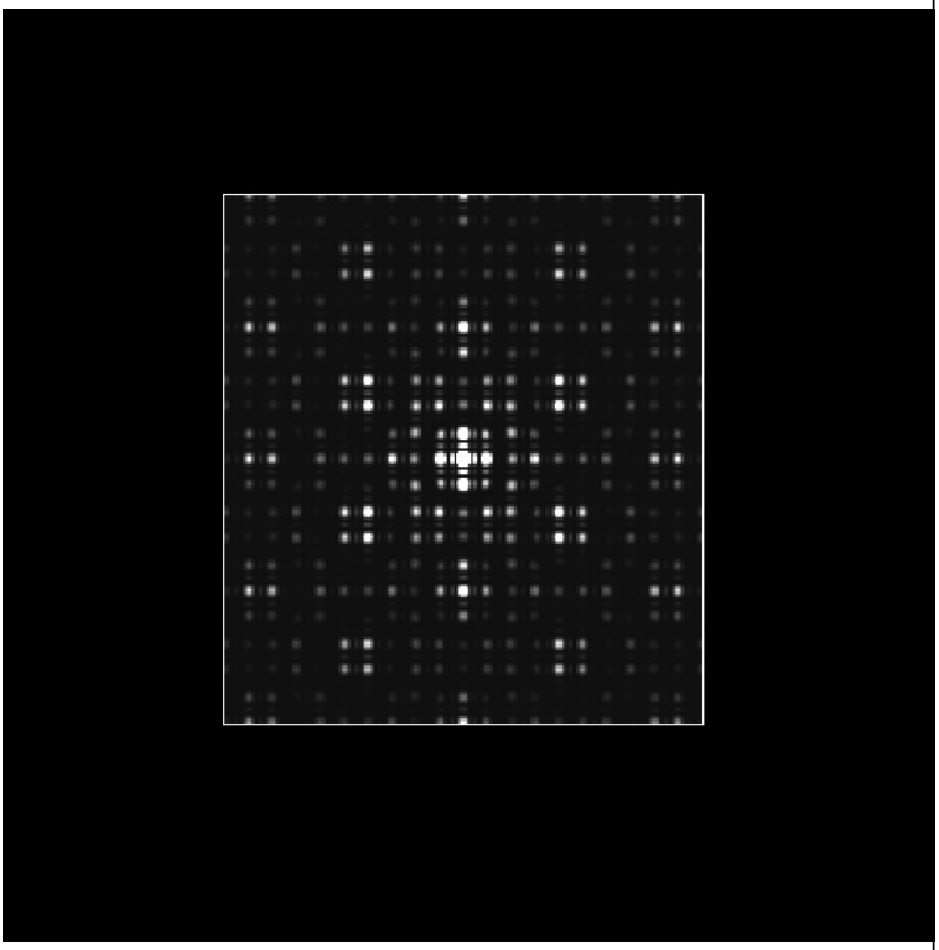



- The size, spacing, shape and orientation of the slit or object are all reflected in an inverse sense in the diffraction pattern

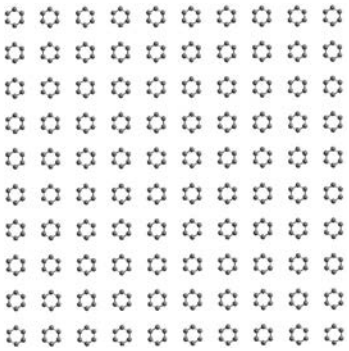


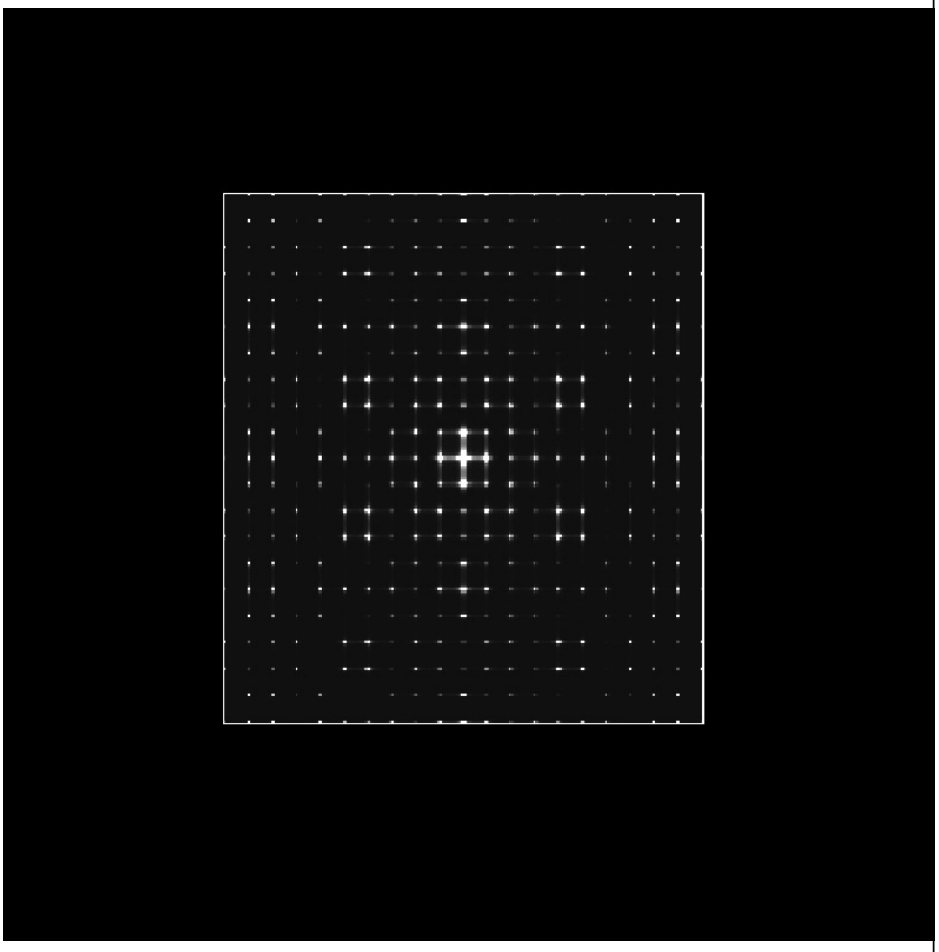




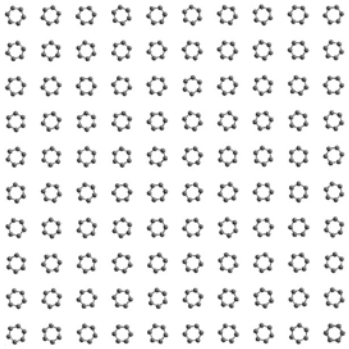


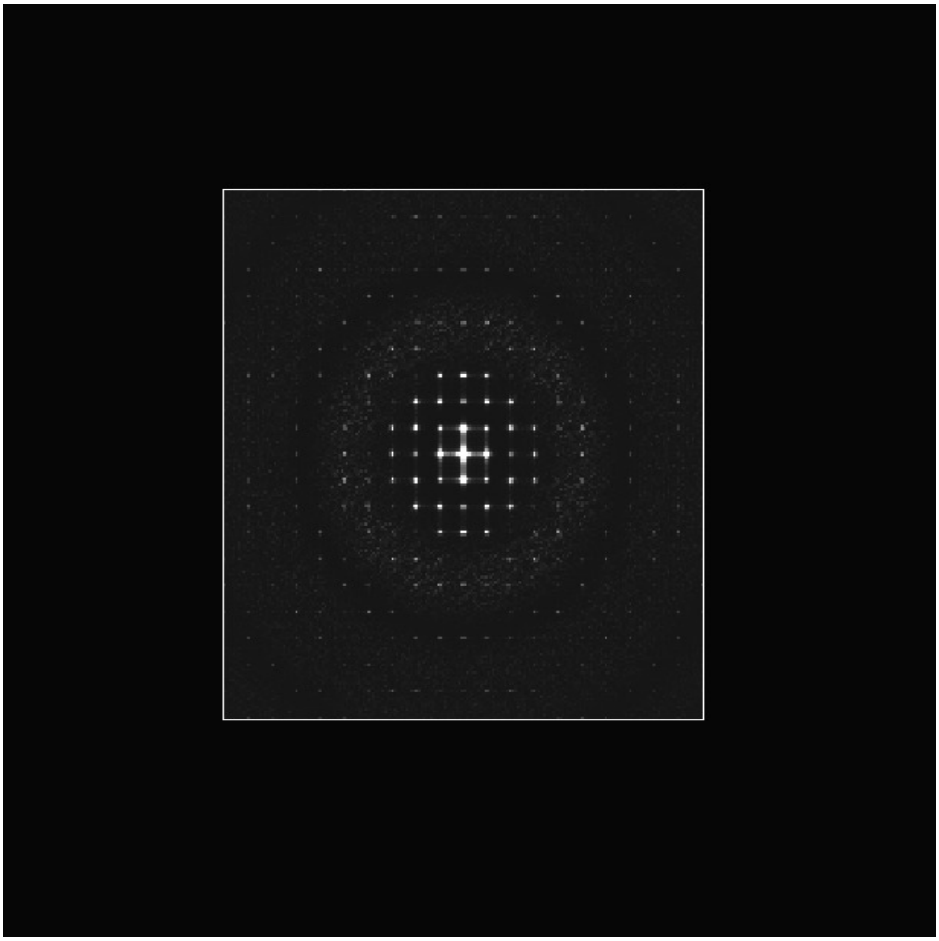
Ordered periodic arrangement
= sharp diffraction pattern (Bragg peaks)



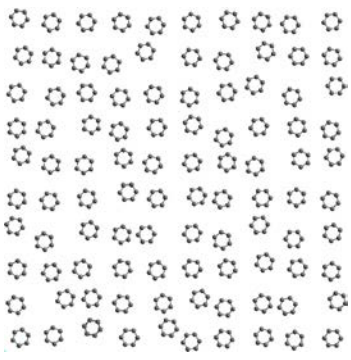


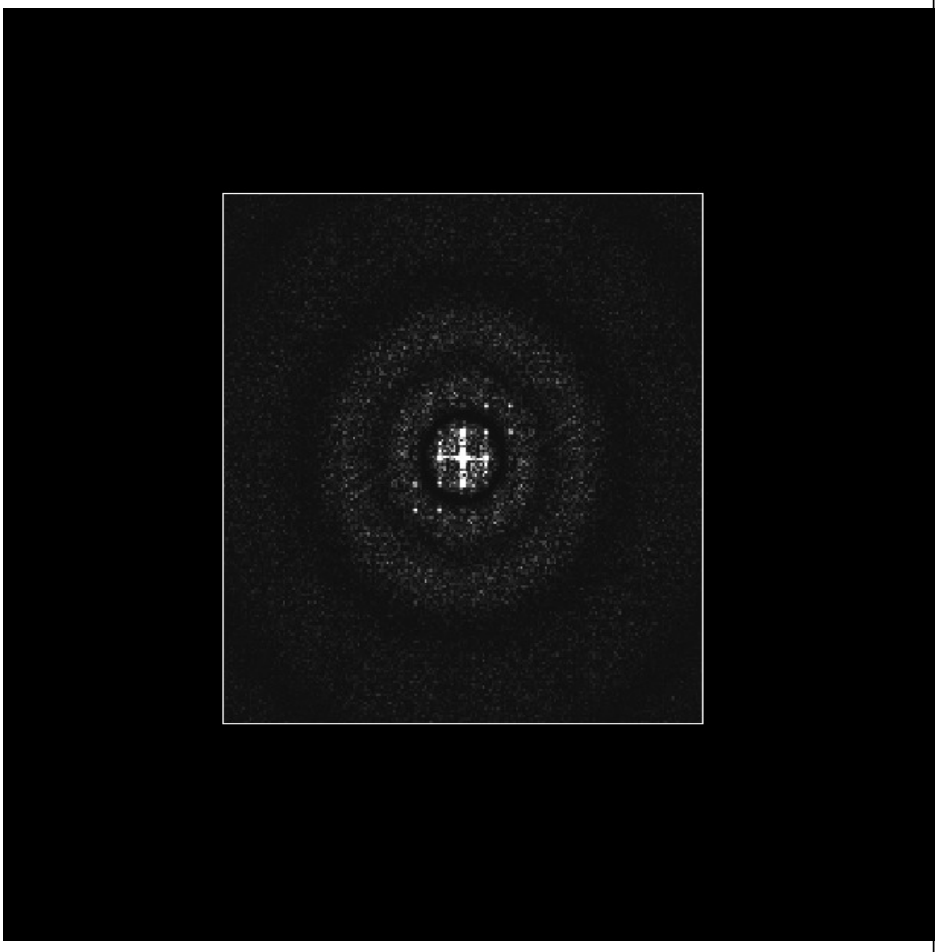
Disorder
= breakdown of the strict periodic array
= reduction in pattern sharpness and in Bragg intensities





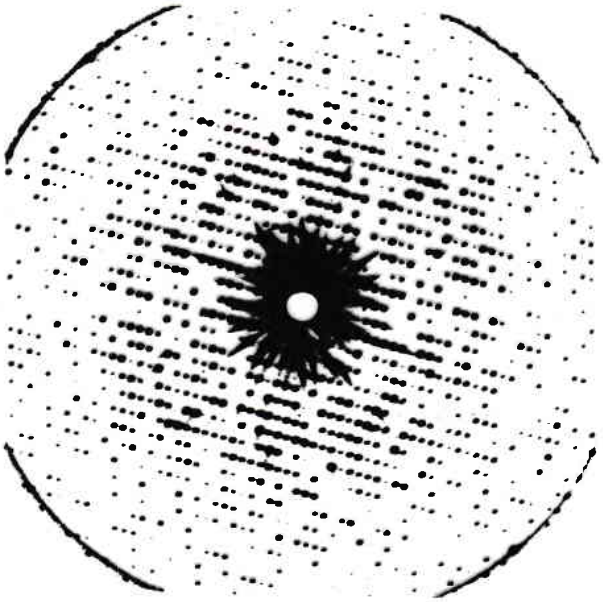
Severe disorder
= loss of periodic array
= loss of diffraction spots and increase in diffuse scattering





Essentially impossible to determine the crystal structure via routine methods

# Diffraction Photograph of a Real Crystal



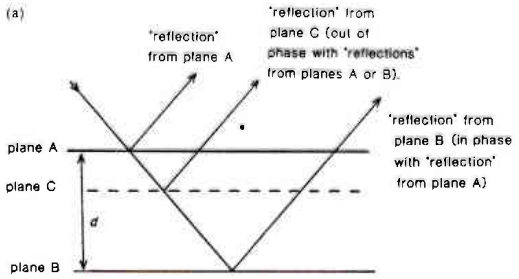
## Bragg's Equation

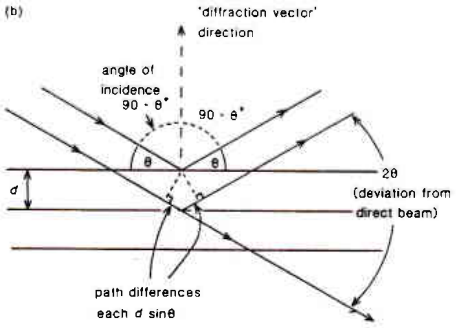
$$n \cdot \lambda = 2d \cdot \sin\theta$$

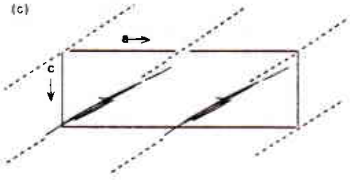
Reflection of X-rays by imaginary planes through lattice points in the crystal

Atoms or molecules on or close to a lattice plane gives strong intensity for the corresponding diffracted beam (reflection)

No atoms or molecules on or close to a lattice plane gives weak intensity for that reflection



When using X-rays, why do Bragg intensities fall off as the diffraction angle increases?

Atoms have a finite size which depends on the element.

Interference arises between the scattering from the electrons at different positions within the atom.

The larger the atom, the greater the interference as the diffraction angle increases.

Cf. single slits of different sizes.

This property is known as the Atomic Scattering Factor and is tabulated for all elements, ions and common X-ray wavelengths.

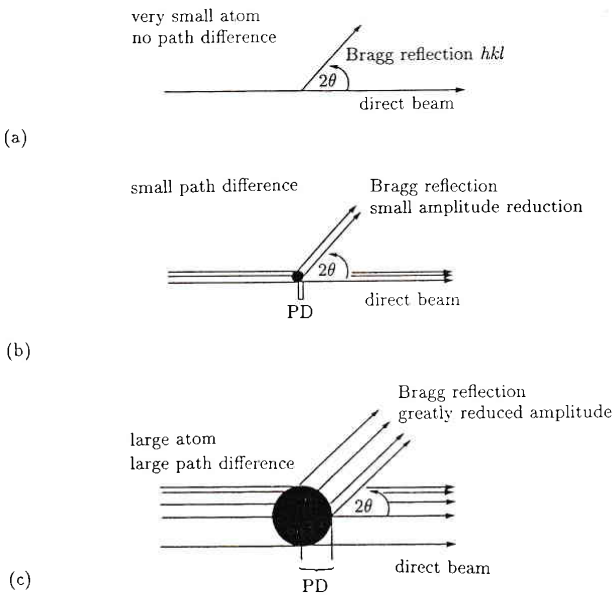
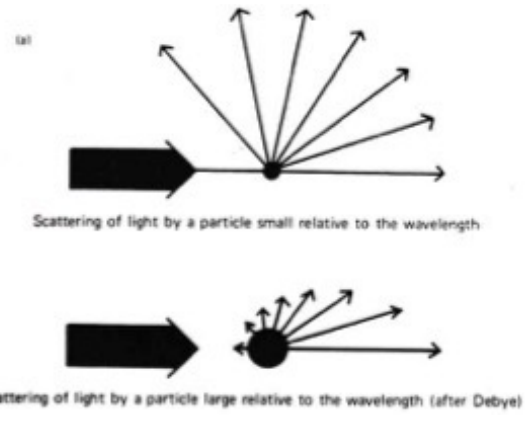


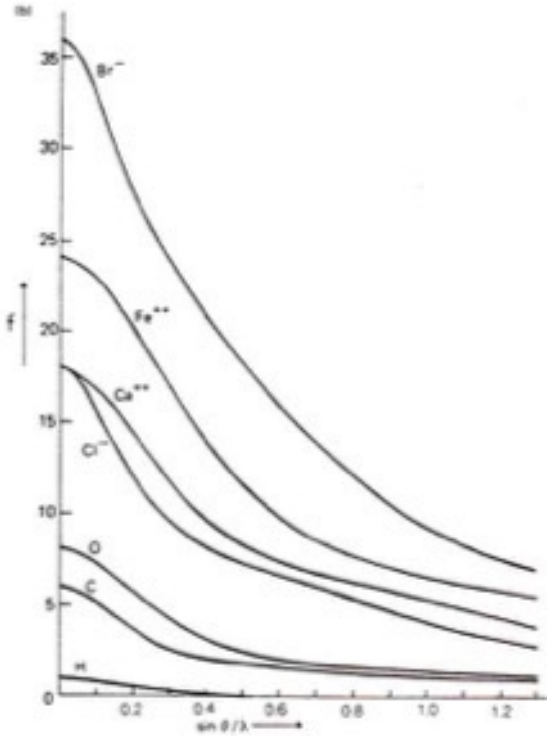

FIGURE 3.12. The effect of the size ment of an atom, relative to the wavelength of the incident radiation, on the intensities of diffracted beams as the scattering angle $2\theta$ increases. As the phase difference between waves scattered by the different outer regions of an atom increases, so does the falloff in intensity as a function of $2\theta$ (or $\sin\theta/\lambda$). (a) A very small atom (e.g., the nucleus of an atom, detected by neutron diffraction). (b) A larger atom, and (c) a very large atom. The falloff in intensity is greatest in (c) because the path difference (PD) for the largest size atom is greatest.

This interference property is known as the Atomic Scattering Factor, *f*, and is tabulated for all elements, ions and common X-ray wavelengths.

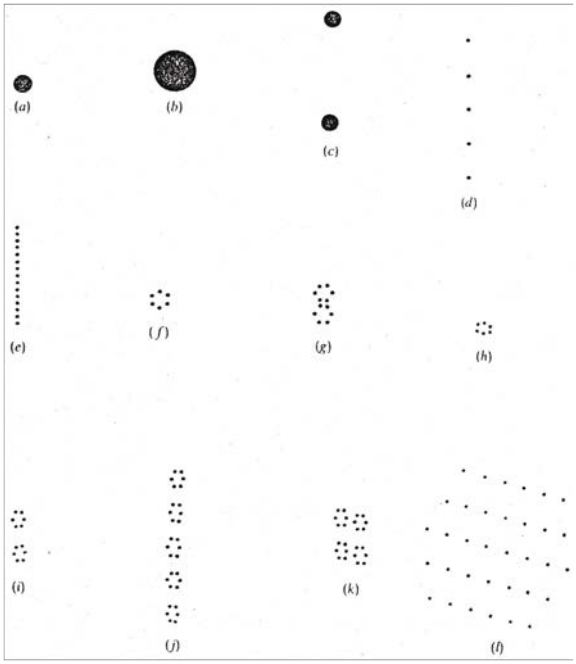
At zero scattering angle (direct beam, *y* intercept on the graph), the *f* value is just the number of electrons in the atom or ion.

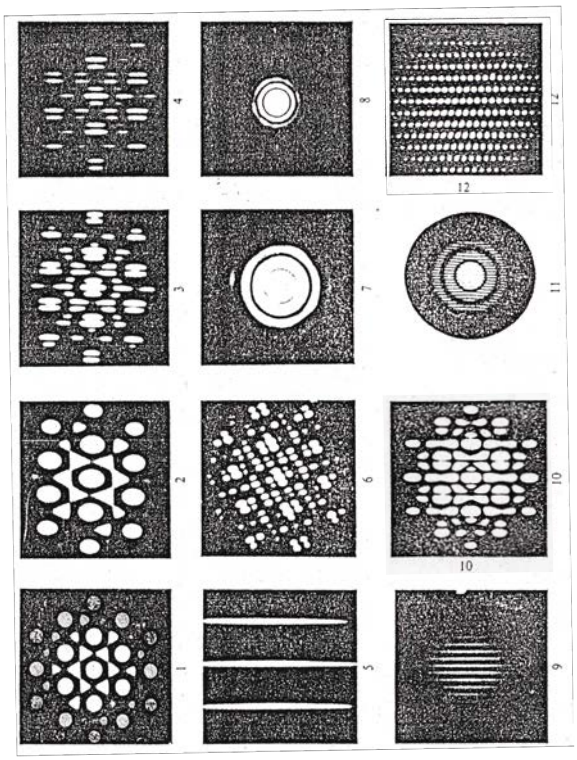
## Scattering Factors



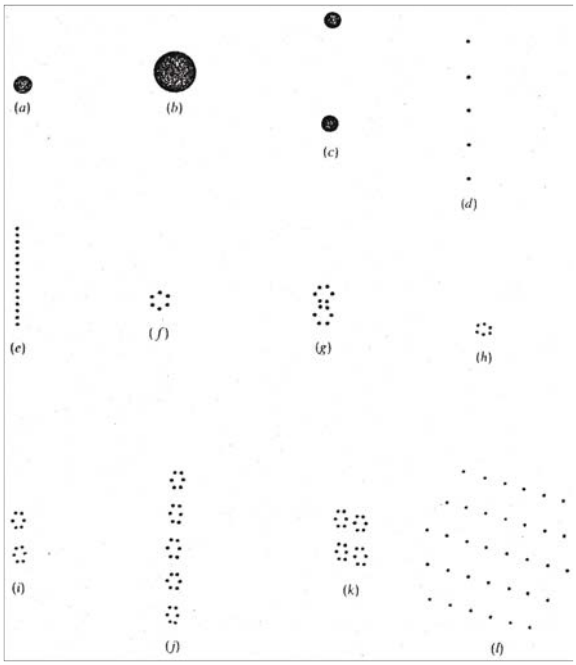



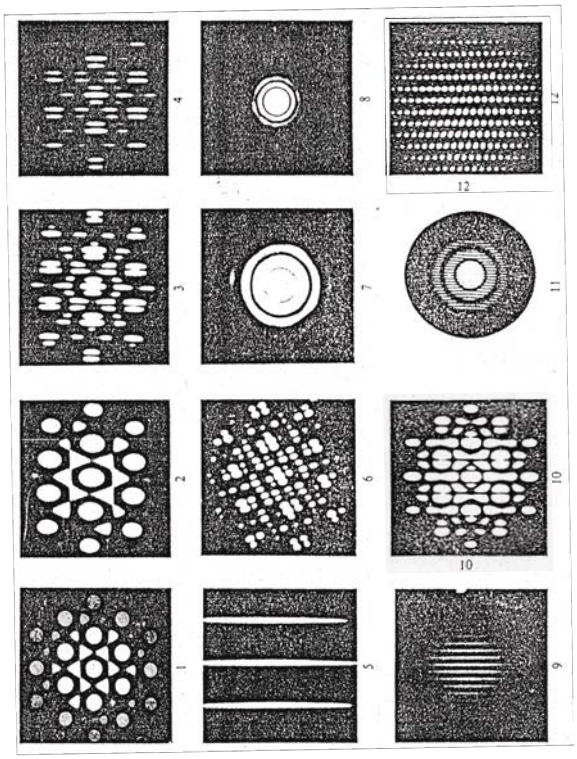
## Match the real space motif to the diffraction pattern



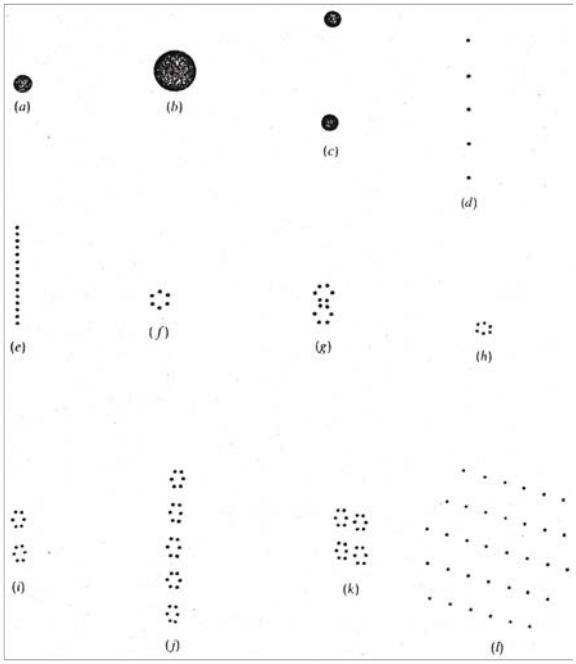

## Match the real space motif to the diffraction pattern

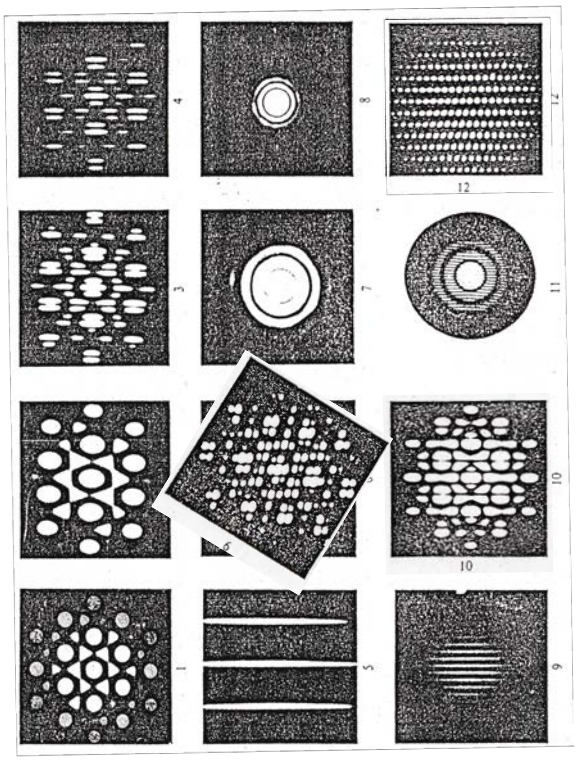



a/7 b/8 c/11 d/9 e/5 f/1 g/10 h/2 i/3 j/4 k/6 l/12

# Match the real space motif to the diffraction pattern




a/7 b/8 c/11 d/9 e/5 f/1 g/10 h/2 i/3 j/4 k/6 l/12; the image for 6 was not given with the correct orientation.
Larger object = smaller envelope (blob); larger repeat spacing = smaller line spacing; more repeats give sharper lines.